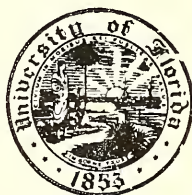


FROM THE PHYSICAL  
TO THE  
SOCIAL SCIENCES


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**FROM THE PHYSICAL TO THE  
SOCIAL SCIENCES**



THE JOHNS HOPKINS UNIVERSITY  
INSTITUTE FOR THE STUDY OF LAW

# FROM THE PHYSICAL TO THE SOCIAL SCIENCES

Introduction to a Study  
of  
Economic and Ethical Theory

BY  
JACQUES RUEFF  
Ancien Élève De L'École Polytechnique

Translated by  
HERMAN GREEN

With an Introduction by  
HERMAN OLIPHANT  
AND  
ABRAM HEWITT

BALTIMORE: THE JOHNS HOPKINS PRESS  
LONDON: HUMPHREY MILFORD  
OXFORD UNIVERSITY PRESS

1929

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THE JOHNS HOPKINS PRESS

*The Ford Baltimore Press*  
BALTIMORE, MD., U. S. A.



*To the Memory of  
My Father*



## PREFATORY NOTE

While one of the final chapters in this book devotes considerable space to a technical exposition of mathematical political economy, which few other than specialists in that field will find useful, this does not detract from the book's general interest and its great value in the current and widespread discussion of the development of the scientific habit of thought and of the logic and philosophy underlying scientific methods. Of wider interest and of even more importance is the book's contribution to the effort now being made in many quarters to borrow from the physical sciences their more exact, objective and fruitful methods for use in the social sciences. This latter is a movement so far-reaching that it may very well come to characterize the history of thought in the period just ahead of us.

So substantial has been the contribution of those who have coöperated in getting this book into English that it is awkward for anyone to make acknowledgments. Chapter XV was carefully edited by Professor Philip G. Wright of the Institute of Economics, and he prepared the valuable explanatory interpolations and critical notes in that chapter. Professor Casius J. Keyser of Columbia University read the entire text and made a great many improvements in it. In fact, it was Professor Keyser's en-

dorsement of it in his little classic "Thinking About Thinking" which led directly to this translation. Dr. Simon Kusnetz of the staff of the National Bureau of Economic Research also examined the entire text and it was much improved by his suggestions.

It is to be regretted that a Preface to this book written by the distinguished French scholar, M. C. Colson, could not be included in this translation.

## INTRODUCTION

As the title indicates, this little book deals with the transition from the physical to the social sciences. It attempts to show that the methods which have proved so fruitful in the physical sciences can be applied to the social sciences as well. Although the emphasis bears on method throughout, yet the philosophic insight is so penetrating and suggestive that almost all readers will be grateful to Mr. Rueff for an abiding addition to the substance of their thought. It was this philosophic view with its application to the social sciences, and to law in particular, that led to this translation, and, has induced law students to undertake this Introduction.

A little reflection may show that there is even further substance in a law student's apology for attempting this Introduction. One is bound to say that there is now no such thing as a science of law unless one is willing grossly to abuse the word "science." And among the principal reasons why there is no science of law is the fact that an approach to its study, of the age and parentage of medieval scholasticism, has kept students of the law going about in circles, largely futile so far as developing a genuine science of law is concerned. This book tells with remarkable brevity and clarity how students of the physical sciences broke out of like futile paths cover-

ing their fields and found ways out and up to more understanding.

What that blighting medieval prepossession is and how this book contributes to eliminating it cannot be better told than in terms of the history of legal scholarship. In no other field of human thought is that prepossession to be found in a more exaggerated and persistent form. That a student of theology, ethics, economics, sociology or anthropology may, as he reads this description of much of our current legal scholarship, be aware of a catalogue of parallel futilities in his own field does not make the picture of legal scholarship less illuminating as an introduction to this remarkable book on the problem of a scientific methodology for the social sciences.

So far as most lawyers, judges and legal scholars are conscious of methods employed in their work, they avow three types of approach to the legal problems with which they deal. For convenience let them here be called the *transcendental*, the *inductive* and the *practical* methods respectively. The first two types purport to be wholly methodical and it is the absence of the methodology thus professed which is a differentiating characteristic of the third type. It is an interesting excursion to take some concrete legal problem of general interest and to watch the application of each of these three methods to it.

It is necessary to use some caution in choosing the problem thus to be studied in order not to miss the whole point to the matter. The problem in a case

before a court for solution may be covered and determined by the explicit language of some valid statute. Again it may be, so far as its facts are concerned, all but wholly identical with some previous case which the court has already decided. This prior decision binds the court under the Anglo-American doctrine of following precedents. In either case the court will decide the question quickly. Indeed, cases whose outcome is thus clearly predestined by some statute or prior decision often have a way of never getting into the courts at all, which is natural enough, and they are not the cases upon which legal scholars and judges spend their time and efforts. Such cases present no real problems at all and are not good cases to study in order to get an understanding of these three prevalent types of method used on legal problems. We need a case presenting some features of real novelty.

Of the all but infinite number of such cases that might be chosen as an example for study, one as good as any other is that which recently arose in Seattle, Washington. A group of the teachers in the public schools of Seattle were members of a teachers' union and others were being solicited to join. When the time came for re-hiring these teachers for the coming year, the Board of Education is reported to have called upon each teacher, as a condition to being re-employed, to sign a contract to the effect that he would not become a member of this union, and, if already a member, would promptly withdraw.

Has the School Board the privilege to refuse to hire a teacher who will not sign such a contract? Suppose this question reaches the court in the form of an appropriate action or suit to compel the School Board to proceed with the hiring of teachers without the imposition of this condition. How should the court decide this case?

Or suppose the matter comes before the court in a different way. Suppose one of these teachers signs this contract but later joins the union in violation of his promise not to do so and is then sued by the School Board for breach of this contract. Is the contract valid in the sense that the teacher is liable in damages for his breach of it?

Assuming that there is no statute applicable to these questions and that no cases all but identical with these two supposed cases have ever been decided by the courts, how is the court to go about solving these two novel problems?

One approach, and, indeed, one most commonly employed, is what is here called the "transcendental" approach. It starts by assuming the existence of some general "principles" within which the solution of these and most concrete problems is hidden away. The theory of this approach seems to be that such a general "principle" can, in some way, be evolved out of one's inner consciousness or sensed as enveloping heat is. Thus obtained, it is then set down as the major premise of a deductive syllogism, the subject of whose minor premise is the case which we are examining. The solution of the problem is



then brought forth in the conclusion which is drawn from these two premises by the operation of the inexorable laws of deductive logic.

One such general principle, which, according to this method, is pregnant with the solution of the first of our two questions, is as follows:

*One who is under no duty to enter into a contract with another may stipulate anything which he pleases as a condition to entering into such a contract.* The appropriate minor premise is: The Seattle School Board was under no legal duty to enter into a new contract with any of these teachers. It follows as a conclusion that the Seattle School Board may impose, as a condition to entering into new contracts with these teachers, anything which it pleases, including, of course, the stipulation that the teachers shall agree not to become or to remain members of the teachers' union.

Similarly, as to our second question, involving the validity of the teachers' contract not to become or to remain a member of the teachers' union, a broad general principle which can serve as a major premise is to the effect that *persons of full age and of normal mental competency have the legal power freely to determine the terms of any contract which they may enter into, and, when they have so determined them, they are bound by the contract which they have thus made.* The appropriate minor premise for the syllogism is: These Seattle school teachers were persons of full age and of normal mental competency. By the operation of the inexorable

laws of deductive logic, the conclusion follows that these school teachers are bound by a valid contract not to become or to remain members of the teachers' union.<sup>1</sup>

One difficulty with this approach is that equally valid lines of argument leading to the exactly opposite result in each case can be constructed. Thus, instead of laying down as a major premise the general principle that one who is under no duty to enter into a contract with another may stipulate anything which he pleases as a condition to entering into such a contract, an equally well authenticated "principle" may be laid down as follows: *Officials administering the trust of public office are bound to distribute the benefits and emoluments of government with impartiality and may not unreasonably discriminate in the appointment of those to discharge public duties and to receive public funds therefor.* The appropriate minor premise to accompany this major one is: To deny employment to a teacher merely because he refuses to agree not to join a particular organization of teachers is an unreasonable discrimination.<sup>2</sup> The wheels of the deductive ma-

<sup>1</sup> If it is remarked that these hypothetical lines of argument are patently fallacious, the answer is that they merely magnify the fallacy, which would be none the less present if less broad propositions were stated as major premises. To avoid such exaggeration by stating narrower major premises would merely make the present discussion more difficult for those not students of the law to follow. It would not, however, eliminate the fallacy inherent in this approach.

<sup>2</sup> Compare a like denial because he refused to agree not to attend a particular college, or to belong to a particular lodge, church or musical society.

chine turn and out comes the conclusion that the Seattle School Board does not have the privilege of refusing to hire teachers because of their refusal to sign a contract not to become or to remain a member of the teachers' union.<sup>1</sup>

Or take our second case as to the validity of the contract not to join the teachers' union. Here again, instead of starting with the major premise to the effect that a sane adult may determine for himself the terms of his contract, we can start with an acceptable major premise to the effect that *freedom of association, whether for social or economic purposes, being one of the primary liberties guaranteed by our form of government, any contract to deprive oneself of this freedom is opposed to public policy and void*. An appropriate minor premise would be: The contract of a Seattle teacher not to become or to remain a member of the teachers' union represents an attempt to surrender his inalienable freedom of association. It follows as a conclusion that his contract not to become or remain a member of the teachers' union is unenforceable.

Upon reflection, it must be clear that, for any case wherein there is a clash of two groups having conflicting interests, two conflicting major premises can always be formulated, one embodying one set of interests, the other embodying the other.<sup>2</sup> Each

<sup>1</sup> People v. City of Chicago, 199 Ill. App. 356; compare Adam v. Brennan, 177 Ill. 194.

<sup>2</sup> See Cardozo's "Paradoxes of Legal Science" for numerous examples. If it be a case involving no conflicting interests the question raised is moot and can be ignored.

group has had its advocate to formulate its interests into general propositions and our novel cases all involve some such conflict of interests.

That two such conflicting major premises can always be found is but the result of the fundamental futility of this approach as a method of determining how novel cases should be decided. This futility resides in the way such major premises are obtained. We "create"<sup>1</sup> them, as this book makes abundantly clear, for the very purpose of serving as the bases of our explanation of that totality of social phenomena which makes up our selected experience, including also what we want to see come about. Of course, they are formulated in such a manner as to include that decision of the case before us which we desire. Those of us who have different and conflicting interests know social "reality" differently. Different factors were operative to determine our respective views of the totality of social phenomena. Moreover, different ones of us desire different things to come about, including different decisions of the case before the court. In consequence, we "created" different causes to account for social "reality" present or desired. These causes so "created" are the conflicting major premises of this method of approach to legal problems. They are the rational, not the empirical side of the shield.

<sup>1</sup> Often the process is one not of "creating" premises but of selecting appropriate ones from the abundance of general formulations that lie about. Their *ad hoc* character is apparent in either case.

This being so, it seems clear that this approach, as a method for ascertaining how any novel case should be decided, is not a certain guide. As Mr. Rueff's book so clearly shows, the deductive syllogism is merely a machine which, if run backward, will produce major premises constituting useful stenographic expressions of the totality of our experience and, for law, this totality means the totality of (*inter alia*<sup>1</sup>) previously decided cases and decisions desired.<sup>2</sup>

Nevertheless, if one is to judge from many of the opinions being turned out by our courts today in the decision of cases, the confidence in the validity of this approach is still wide-spread. This confidence is rooted in an articulate or inarticulate belief in a "natural law" made up of abiding "principles" of right and justice whose existence transcends change in time, place and circumstance. There is neither space nor occasion to write here the history of this school of thought but it is worth while to note how persistent and vigorous it still is in the field of law at least.<sup>3</sup>

<sup>1</sup> *E. g.*, statutes. It would merely complicate the discussion to carry along parallel treatments of such other factors.

<sup>2</sup> States of fact desired appear in the empirical propositions subtended by the postulates of some of the other social sciences and of the physical sciences as they shade off into such bodies of knowledge as engineering and horticulture, for examples.

<sup>3</sup> That making the case to be decided the subject of the minor premise gratuitously reformulates the major premise, or, more exactly, redefines the middle term, is another way of stating that deductive logic is sterile as a source of new empirical knowledge.

The second method of approach to the solution of legal problems is what has here been called the " inductive " one. This approach, like the one already described, employs the deductive syllogism, in which syllogism, as before, the case to be decided constitutes the subject of the minor premise. The outstanding difference between the two approaches has to do only with the theory as to the source of major premises.

The " inductive " approach does not assume a system of fundamental and changeless general " principles " existing apart from cases decided in the past. It purports to derive them from an examination of a number of such particular decisions.

Any one of the four hypothetical major premises already set out will serve to illustrate how this is done. Consider the first, which is to the effect that one who is under no duty to enter into a contract with another is at liberty to stipulate anything which he pleases as a condition to entering into such a contract. The method of inductively deriving such a premise consists of examining a number of cases, in which persons other than teachers entering into contracts relating to matters other than union membership have been held free to stipulate contractual terms of varying content. Cases thus examined will include other stipulations insisted upon both by persons not in public office and by those in public office. Such other stipulations will relate to matters of more or of less vital concern to the other party than



a teacher's freedom of organization is to him. The cases so examined may thus have the widest range as to their practical similarity to the case up for decision. From all of these cases there is "induced" a principle which becomes the major premise of a deductive syllogism, the subject of whose minor premise is the case to be decided.

If the principle thus "induced" is no broader than the sum of the previous cases which it summarizes, it obviously does not and cannot include the case to be decided, which, by hypothesis, is a new and an undecided case, and, hence, can form no part of the generalization made from previous cases only. If it does not include the case to be decided, it is powerless to produce and determine a decision of it. If it is taken to include the case to be decided, it assumes the very thing that is supposed to be up for decision.

Just as was true with reference to the first approach so also as to this second approach, for each general principle induced from one set of cases, a conflicting general principle leading to an opposite result can usually be "induced" by selecting a different group of past decisions to serve as the basis of the induction and the same variety of factors governs that choice.

To sum up thus far, both of these two approaches have these two weaknesses in common. Each can be used with equal validity by both parties to the litigation and thus be made to "prove" exactly oppo-

site propositions. Each approach must assume that the major premise is broad enough to cover the case to be decided.<sup>1</sup>

Both of these two logical approaches, the "transcendental" and the "inductive," beg the question they are set to solve, and are, therefore, inadequate for the solution of new cases. In each case, the decision reached will depend on the major premise adopted. This in turn will depend upon which of two conflicting interests is to be served.

This book suggests two examples of this type of thought which it is interesting to juxtaposit. At the opening of the "Politics," Aristotle justifies slavery, then an accepted institution, on the ground that most men are slaves by nature. In sharp contrast is the social order which Rousseau envisaged in the "Social Contract." He wished to proclaim a social order wholly different from the existing absolute monarchy in France. The opening sentence of his book, in consequence, states that man is born free, but is everywhere in chains.<sup>2</sup>

<sup>1</sup> Stated otherwise both approaches have the weakness of assuming that the case to be decided falls within the minor premise, which automatically redefines the major premise by being placed under it. This is true whether that major premise was assumed or was validly induced from previous cases.

<sup>2</sup> And so it is today, the human mind seeks to comfort itself by blanket phrases in the form of causal explanation which, on analysis, mean little. For example, we say we punish criminal conduct because the accused was "responsible" for the offense. If he was irresponsible as when insane, that defence prevails. And yet we mean by responsibility no more than that there is for us no assignable cause beyond the author of the conduct.



Under the guise of logic, then, we have methods purely arbitrary, everything depending on the choice of the major premise. This is not objectionable as method; the abuse lies in applying logic in the proper sphere of the empirical. When so applied, there is nothing to insure that the major premise chosen bears any useful relation to prevalent social values—the essence of justice. It is quite as likely to be the dogma of a medieval ghost still ruling us from the mists of antiquity.

This criticism applies to all arguments. So far as proof goes, every argument in and of itself begs the question it is set to solve. The classical “proof” that Socrates was mortal exemplifies the point. As its very first step, the conclusion was implicitly assumed for, as soon as we say that all men are mortal, Socrates is implicitly included. This example illustrates the proper function of argument; not to prove the conclusion desired, but to make explicit what is already implicit in one’s position on a question.

Such a logical process being perfectly automatic, new knowledge must depend primarily on the phe-

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For any causal explanation there must be two terms. There is only one here, namely, the author of the conduct whom we label “responsible.” Therefore, no causal explanation can be found for such conduct.

And, for the purpose of studying criminal punishment, criminal conduct extends as far as, and no further than, responsibility. Therefore, to say that we punish criminal conduct because it is “responsible” conduct is to say that we punish criminal conduct because it is criminal conduct. The tag “responsibility” acts as a comfort to the frail human mind, but throws no light on why we do what we do.

phenomena selected for correlation and explanation. As soon as this is clear, it becomes imperative to agree upon what the phenomena are. It is precisely the extent of this agreement which accounts for the degree of solidarity of thought in the natural sciences, and it is the lack of such agreement which so largely explains the rank confusion in the social sciences. True it is that phenomena of the order or range which scientists have sought are far more easily observed in many of the natural sciences for therein they are more often dealing with objective ponderables and measurables, but, in the social branches, no comparable systematic attempt has been made to concentrate upon an order or range of phenomena enabling us to deal impersonally, and, so far as possible, objectively with factors hitherto regarded as necessarily subjective and imponderable.

This fact serves to illuminate many well-known controversies in such fields as morals where contestants have spent many laborious and strenuous years in picking logical flaws in the arguments of opponents. A close scrutiny will reveal that, in most cases, there was no agreement upon what the phenomena to be explained were. The disputants were not talking about the same things. Of course, the logic of an opponent talking about a wholly different set of phenomena was absurd because it was not designed to explain the phenomena that the critic wished to see explained—neither party, however, realizing any disparity between the two sets of phenomena, each tacitly assuming them to be identical.

It may well be that substantial agreement upon social phenomena can never be reached. In that case, dispute and controversy will continue to thrive, and little certain advance can be made. From this it might be argued that the method which is valid for the natural sciences will be invalid for the social sciences. Here lies the crux of the matter and the significance of Mr. Rueff's book. He has developed the idea that explanation by the Law of Causality, or the causal explanation, is the only one which the human mind understands.<sup>1</sup> He has shown that the process of explanation proceeds from the observation of phenomena to the "creation" of causes, from which can be deduced laws corresponding to the empirical laws.<sup>2</sup>

<sup>1</sup> Apart from more fundamental difficulties, there is an ambiguity in this notion of "cause." It is often thought that the relation of cause and effect is solely that of a creation of the effect by the cause. This may be largely due to the fact that effect is thought of as following the cause in time. Using the words in a broader way, "causal explanation," applies also to things such as mathematical quantities, which are thought of as being in a fixed relation, and not in temporal sequence. In this sense, "causal explanation" deals with the fixed relation between things, and also more commonly with their sequence in time.

<sup>2</sup> It is to be hoped that Mr. Rueff will at some time elaborate upon the relation between the rational and the empirical. He speaks of the rational process explaining the empirical law observed. It is obvious he cannot mean that what we observe is the law; we observe phenomena. As he points out, the very assertion of a law implies that the phenomena have been selected and correlated by an intelligent process. And what is this if not rational? It may be that the rational process he speaks of as distinctively rational deals not primarily with the empirical but with propositions, or the formulation of propositions from which laws can be deduced identical with the

The only argument which can be advanced against this position leaves it undisturbed. For, even though it were admitted that such a method will not prove helpful in the solution of social problems to the same extent as it has succeeded in the solution of mechanical problems, nevertheless, if it is the only type of explanation which the human mind can understand, it follows that the utmost advance possible by explanation, and perhaps any advance, will be made by that same so-called scientific method.<sup>1</sup>

In the light of this argument, it would appear that the limit of advance in the social sciences will be set by the fruitfulness of the ranges of phenomena we select for observation and by the exactness with which such phenomena can be observed. There is every reason to suppose that our technique of observation in this field will improve with the necessity of using it, just as it has improved beyond reasonable hope or foresight when put to work in the field of the physical sciences.

Setting out the underlying logical method for the social sciences and showing that it must be the same in kind as that of the natural sciences constitute the great contribution of Mr. Rueff's book.

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empirical laws. Yet it would seem as though reason, as opposed to mere observation, was at work in both processes, one of which has a purely empirical base, and the other a purely logical one.

<sup>1</sup> Of course there may be other types of experience, such as the religious and the intuitive, which do not fall within the scope of causal explanation. The above statement is restricted to conscious and systematic processes of thought.

To return to the methods of approach to legal problems, the third is the one here called the "practical" approach. Typically it is resorted to only when there arises a case for which no appropriate major premise is apparent (either as a matter of sheer assumption or of induction from prior decisions) or for which evenly competing major premises are so obviously present that this fact cannot be ignored. When so driven to it, the orthodox technique of the judge is to consider the question, as he says, "not on principle but on policy," *i. e.*, explicitly to consider which way, as a practical matter, the case ought to be decided.

When this third method is resorted to, there is seldom any informed and exhaustive marshalling of the practical considerations *pro* and *con*. The court typically reaches its practical solution by reliance upon "common sense"—, a sort of intuition of experience which assumes to know how to decide the practical questions of life merely as a result of having lived in life. In exceptional and rare cases where the judge's "common sense" experience is quite obviously inadequate because of the technical character of the question in issue, resort is had to the expert judgment of others as to the practical effects of deciding the case one way or another.

It is, of course, true that to apply to a novel case, the first and second approaches already discussed means that that case must be decided by the operation of sheer chance or by the operation of practi-

cal factors. It is really the court's conscious or unconscious consideration of the latter that determines which major premise shall be chosen when either of the first two methods is being pursued. In practical effect, therefore, the difference between the first and second method on the one hand and the third method on the other is that the weighing of practical considerations is *consciously* done in the latter only. It is not *methodically* done in either.

If all of the foregoing is considered in the light of Mr. Rueff's discussion, it will be seen that, so far as law has an empirical branch, its precarious existence is largely either unsuspected or is the haphazard product of a "common sense" empiricism which professes no order and no methods. But the rational branch of law is not meager or formless. Most of the literature of law, whether found in judicial decisions or in academic writings, is devoted to erecting, defending, or attacking elaborate structures of autonomous thought which seek to describe observed and desired reality by a series of inverted pyramids of abstract propositions (each successive proposition being broader than the last), the reality in question being the actual decisions of cases by courts.

As Mr. Rueff so forcibly points out, the development of the rational branch of a subject accomplishes many important things in the advancement of knowledge. Therein reside indispensable mnemonic devices and means of communication. Among other things, the operation of the deductive machine



formulates many new propositions, the truth of which it is the business of the empirical branch of the subject to test by observation or experimentation. For such a rational branch to be productive in this latter respect, however, it is necessary that its basic assumptions or postulates have some measure of current significance. Little of current significance attaches to great masses of the law's present postulates, which originated in medieval scholasticism and which have been subjected to the rigors of little deliberate empirical testing since that time. When, as is true of the second approach to law, the premises or postulates are arrived at by "inducing" them from prior decisions, they are somewhat more promising but often of surprising antiquity. Moreover, a misplaced confidence in the power of logic, whether deductive or inductive, in some mysterious fashion, to coerce the human mind into sure paths leading to sound answers to novel practical questions has resulted in an all but complete absence of any testing of conclusions by observation or experimentation, even by those who scorn "natural law" and profess rigidly to follow inductive processes in passing upon new cases.

There has been such a complete absence of effort methodically to develop the empirical side of law and such an over-elaboration of its rational side that scholarship in law tends more and more to neglect how courts actually decide cases and more and more to consider what they say about why they decide as

they do, which, after all, is stating the same thing in another way.

Unless, therefore, one is prepared to call a body of learning which has substantially no empirical branch or techniques a science, there is no science of law. But how there may be, is made abundantly clear by Mr. Rueff's little book. Part of his contribution is the clear fashion in which he has pointed out how the natural scientist "creates" causes, just as the social "scientist" does and must do and that there is no difference in the nature of the subject-matter of law or of any other social "science," as contrasted with physics, for example, which precludes a like rigid scientific pursuit.

That pursuit will yield most after a generation of students of the law foregoes the temptation to continue elaborating existing assumptions and relying on a "common sense" pragmatism in its empirical testing and instead devotes itself to an effort to arrive at assumptions of greater current significance by building them upon observations of contemporary social reality, using in such observations methods as impersonal and objective as possible.

It is common for those despairing of the development of the empirical branch of law by the widespread application of scientific methods in legal study to assume that methods having the objectivity and precision of those of mathematical physics are a prerequisite to our having any scientific methods in the study of law. Reflection reveals, however, that



objective methods having great utility in such areas as biology and geology differ widely from the methods of mathematical physics with respect to the degree of their precision and objectivity.

The word "objective" in the term "objective method" is relative and pragmatic. A very gross measurement of an object may be sufficiently accurate for the purposes of a rough carpenter and hence for his purposes "true," while hopelessly inaccurate for the purposes of a physicist working upon the expansion coefficient of a metal. The personal equation and subjective elements bulk large in the first measurement, but, for the purposes of a carpenter, they are unimportant. They are, of course, present in the measurements of a physicist. All that he can hope to do is to reduce them to a minimum, and how far he needs to reduce them depends upon the purpose of the particular measurement. All this is equally true of objective methods in legal research. When this fact is fully appreciated by legal scholarship, its hope for truly scientific methods may be found to be nearer to realization than it is now thought to be. Other social scientists are certainly making substantial progress in developing objective methods in many fields. And, in consequence, some of these other social disciplines are beginning to develop empirical techniques and rational branches worthy of the name science.

A word may be ventured concerning the philosophical position of the author and regarding his use of terms.

The book avoids the eternal metaphysical question of ultimate reality as will appear from a quotation from it. "Thus, to these two questions, namely, the validity of our logic for the external world, and the exact representation of phenomena by the propositions which translate them, the classical (that is, the subjective) conception of physical reality does not permit of a reply. We shall see further on, that these questions do not come up, and are, in fact, meaningless."

The last sentence quoted gives the central position of the book on this metaphysical question. However, certain passages and terms suggest that the author oscillates between a wholly subjective and an objective theory of reality. These passages are due, not to a real deviation from the central position, but to the poverty of our philosophical vocabulary to convey ideas without implying metaphysical notions which had no place in the author's mind.

The question of truth is dealt with on a subjective basis, not because that represents ultimate reality, but simply because it is the inescapable point of view for the human mind. The author shows that we know no more of the matter than that the human mind works according to the laws of Identity and of Causality. "But it is indispensable to note that truth thus defined derives its entire existence from a form of our minds. It is essentially relative to us and has meaning only for us. Outside of ourselves, there is not, nor can there be any criterion of truth."

If this point of view is kept in mind, Mr. Rueff's use of terms, otherwise difficult to follow, becomes clear. He often speaks of "physical" theory as a typically scientific theory, and then, almost in the same breath, states that the human mind "creates" the causes underlying such a theory. Throughout the book, the expression "creation" of causes is used. Mr. Rueff means no more than that we formulate assumptions from which an orderly description of phenomena can be deduced. He does not mean that a new objective independent entity is created by the mind. As we read him, we are not moving in the realm of metaphysics and metaphysical implications should be rigidly excluded.

The "physical" is not the objective, but that which reduces itself to "an axiom which permits the reasoning machine to interpret rationally the succession" of phenomena. Likewise with facts; "to enunciate a fact is almost to create it entirely." At another place he will say, in effect, that such facts are no more than the phenomena furnished by our sensations.

In the light of this use of terms, it can be seen that it is quite possible to pursue a rigidly scientific method without the materialistic metaphysic which usually accompanies it. The question of method and the question of metaphysics are wholly distinct. This book is occupied with the question of method. And it is one of the book's chief merits that it makes

clear how this method can be used in conjunction with any belief as to ultimate reality.

To speculate upon the author's position if he were pressed upon the question of the theory of knowledge, he would probably take a position, midway between scepticism and transcendentalism, to the effect that our minds supply the causal connection whose existence scepticism denies, and, within the realm of our experience, the assumption of such causal connection continues to serve us well. Until experience indicates it is untrustworthy, why abandon it? Similarly, why go beyond the phenomena present to the mind into the lofty realms of transcendentalism? It is just as foolish to assert such a reality beyond our experience, with the transcendentalists, as it is to deny it with the sceptics. Mr. Rueff would probably stand on middle ground, asserting our consciousness of phenomena that are connected by "causes" supplied by the mind,—no more, no less.

HERMAN OLIPHANT,  
ABRAM HEWITT.

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# FROM THE PHYSICAL TO THE SOCIAL SCIENCES

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## PART ONE

### GENERAL CONSIDERATIONS

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#### CHAPTER I

##### INTRODUCTION

Reading a book on ethics is, for a scientific mind, a source of great astonishment. The systems are innumerable but the conclusions are alike. Whether the system of ethics treated be religious or utilitarian, whether it be finalistic or pragmatic, it leads invariably to an ensemble of very precise rules, which are, with very few variations, a humanized code for our civilized world.

As for the method which evolved these laws, it is almost always peculiar to the social sciences. The line of reasoning begins with concepts which are not very clearly defined. It invokes hypotheses which have not been enunciated, and all too often, the true, the beautiful, the good—undefined or undefinable entities—interpose at each step to lead it wherever one may wish it to go. The bewildered scientist receives the impression that the process of proof is only accessory since it always leads to a conclusion which he knows beforehand.

Most of the books on political economy present the same general character, although to a lesser degree.

And yet the tool that built these sciences is the very same one that created the mathematical sciences, viz., the mind of the geometrician, that of the physicist. Why, then, this chasm? Why does the geometrician feel that he steps on quicksands the moment he enters the realm of the moralist? If the rules of morals are one, why are the systems of ethics without number? This is the problem to which we here address ourselves.

We have studied the fundamental nature of the social sciences and it seems to us that we have discovered its essence. We have compared it with that of the clearly "scientific" sciences. The relation between the two, which is the subject of this book, has led us to a series of conclusions. There is not one science, but sciences, all of the same nature, all utilizing a method which is rigorously unique—to wit, the scientific method. Of this geometry furnishes the perfect type. All sciences are rational sciences like geometry, and, since there is a deep-seated similarity between the social sciences and other sciences, there is nothing to prevent us from presenting ethics and political economy under a rigorously scientific form. Thus set forth, ethics becomes a rational science, just as geometry is, deriving all its uncertainty from this very identity.<sup>1</sup> At the same time, the fact

<sup>1</sup> This is neither a misprint nor a paradox. The following pages will explain it.



that there are so many systems of ethics and that their conclusions are the same will be accounted for.

When more broadly viewed, these comparisons will disclose to us the true nature of the social sciences. They will, in addition, lead us to formulate a number of practical rules to guide us in the study and exposition of these sciences.

## CHAPTER II

### THE METHOD WHICH WE SHALL FOLLOW

The goal we have set for ourselves imposes the method we must follow. We wish to bring out the profound parallel which exists between the social sciences and the sciences properly so called. We shall accordingly seek out the characteristics of each of these two kinds of sciences. Their juxtaposition will then show us their underlying similarity.

We shall first study the "rational ego" which builds up the sciences and we shall then attempt to reduce it to a few simple laws. Then we shall pass on to a study of the tools which it has forged and which constitute for us the "reasoning machine." This "machine" consists of: first, syntax or the aggregate of rules governing the association of words; second, formal logic or the aggregate of rules governing the association of propositions; third, mathematical analysis or the aggregate of rules governing the association of a particular category of words representing magnitudes.

The tools once studied, we shall examine their use. This will be done in the study of first, the physical sciences, *i. e.*, geometry, mechanics, astronomy, physics, and chemistry; and second, the social sciences, *i. e.*, psychology, ethics, and political economy.

The fruit will then be ripe. We shall need then only to bring the two groups together. The parallel will be apparent and we shall draw therefrom all the consequences which it is possible to draw.

## CHAPTER III

### THE RATIONAL EGO

All the judgments which we make about the world imply a recognition of two great laws which seem to dominate our entire intellectual life. These are first, the Law of Identity or of Non-contradiction—A is A; and second, the Law of Causality which requires that every phenomenon must have a cause and that the same causes must always produce the same effects.

The very expression of these laws hides their generality. They rule everyday knowledge as well as scientific knowledge, but in a hidden fashion and so vaguely that it is not always easy to detect their presence. We cannot conceive of a sane mind refusing to submit to these laws; and yet the results to which our study of physical theories will lead us will show us how absolute a value it is fitting to accord them, what the character of their necessity really is.

What must be noted and kept in mind is that these laws are fundamental to all scientific study. Since man first began the systematic study of the universe, he has been unable to think outside of these laws. This is a material and indisputable fact which has determined “scientific” form. It is necessary to distinguish this historical fact from the question of the origin of these laws, which falls outside the scope of this book.

The Law of Identity alone suffices to form the basis for deductive reasoning. The syllogism, which is its type-form, can as a matter of fact be reduced to the statement that what is true of the group is true of each individual component thereof. Here we have an immediate consequence of this law which forbids thought to contradict itself.

Inductive mathematical reasoning or reasoning by recurrence can, in our opinion, be justified in like manner by the Law of Identity. It is not, after all, more than just an infinite series of syllogisms. Now is there any difference between stating that the mind can repeat the same operation an indefinite number of times and stating that the mind is consistent with itself?

Induction in physics, on the other hand, can be legitimized only by a new law, the Law of Causality, which affirms that every phenomenon is determined by an antecedent called its cause and that the transition from the cause to its consequence takes place in accordance with an immutable law.

It is important to dwell upon the great difference which is commonly thought to exist between the Law of Identity and the Law of Causality. It is said that the first is only a law of our own minds, while the second purports to link things together, to govern the concatenation of external realities. Now this difference is only an illusion. We shall see further on <sup>1</sup> that the Law of Causality governs the succession of

<sup>1</sup> Chap. VII and VIII. Geometry and Mechanics.

phenomena only when we express them with the aid of certain conventions, our methods of measuring, for example. These conventions have been chosen for the sole purpose of reconciling the succession of our perceptions with this Law of Causality, our brute sensations refusing to yield thereto. Thus explained, the Law of Causality, which seems to be the expression of physical reality independent of ourselves, owes its existence merely to the effort of our mind to impose that law upon the world.

This point once admitted, it will be easy to demonstrate that, if the mind wished to see causality ruling over the world, it was only to render intelligible the system of our ever-changing sensations by regarding them as the multiple manifestation of some profound reality always identical with itself. This reality is merely the aggregate of immutable laws which our experimental physics reveals to us, and the mind is thus satisfied, having discovered identities beneath the immense flux which surrounds us. The Law of Causality is a device which we unconsciously require in order to reconcile our sensations with the Law of Identity.

The bases of our intellectual activity may thus be reduced to the Law of Identity alone. Our minds will hold as true whatever involves no contradiction.

But it is indispensable to note that truth thus defined derives its entire existence from a form of our minds. It is essentially relative to us and has meaning only for us. Outside of ourselves, there is not, nor can there be, any criterion of truth.

## CHAPTER IV

### THE REASONING MACHINE

The reasoning machine is the aggregate of laws which permits the formulation of rational structures in accordance with the Law of Non-contradiction.

It includes two distinct mechanisms, or more exactly stated, two distinct adaptations of the same mechanism. These are: Formal Logic, or the mechanism for reasoning in terms of propositions, and mathematical analysis, or the mechanism for reasoning in terms of magnitudes.

With primitive man, the reasoning machine reduces itself to a mechanism for the association of ideas. Thought flows on indefinitely, completely at the mercy of haphazard associations. One idea calls forth another which superimposes itself upon the first. Each one derives its existence from all those preceding without any possibility of separation. The stream of thought is a complete whole, composed of an infinite number of elements, and, consequently, it can be neither fixed nor described. Peter is unable to know the thought of Paul for, if he did, he would coincide with him—Peter would be Paul.

So, it was found necessary to socialize this train of thought, which had been essentially personal. Ideas had to be translated into a medium of exchange in order to be made current. Accordingly, just as

we are obliged to cut up a mountain into small parts when we wish to move it, so we were obliged to cut up the current of thought into simpler elements. From these words were made. Each word became a small parcel of thought, impersonal and therefore universal. Impoverished and very rigidly defined, discolored and insipid, the word was convenient and precise because of its weakened content. Thanks to it, it became possible to create the same idea in several minds at the same time.

Once the word had been invented, then in order to give back to the image it evoked some of its primitive richness, it was found necessary to supplement it with other words. But in order that this combination of words might also be rendered universal, and thereby intelligible to individuals who had not previously exchanged conventions, a certain number of types of combinations became defined, these types bearing as close a resemblance as possible to those which the unhampered mind, thinking without words, habitually employs.

Thus willy-nilly, thought, wishing to make itself understood, was obliged to submit to rules of syntax, to pass through a mold which rendered the interchange of ideas automatic and sure. Thenceforth, there was little room left for initiative. If I wish to say a certain thing, syntax dictates the way I must say it. I am thus sure of speaking a language which is intelligible to my interlocutor.

It is necessary to note that the rules of syntax thus understood are entirely superficial. They do not



penetrate to the very depths of my thought. They only fix its form. If I wish to give expression to a self-contradictory statement, they do not prevent me from so doing. They merely give me the assurance that the idea to which my words give rise in my listener's mind is the one I intended he should have.

But thought is subservient to the Law of Non-contradiction. The observation of this Law is the one condition essential to the functioning of our minds.

There is no need, I think, to inquire into the nature of this law. The problem as to its nature exists only when it is poorly stated, *i. e.*, when we imagine in each of us one being which thinks and another which watches the other being—a sort of meticulous guardian of the constitution of our minds, rigidly rejecting all contradictory thoughts. It is our philosophical vocabulary which is responsible for this error. In naming consciousness and stating the Law of Non-contradiction, it gave them a sort of existence independent of the thinking “ego.”

The reality of the case is quite different. Our faculty of setting forth judgments exists independently. We have it as it was given to us and the Law of Identity is nothing but an abstract rule which states *a posteriori* the general character of all our judgments. Now, it is a fact that only those judgments are “human” which contain no contradiction. The statement that a thing is, at one and the same time, man and not man is only black marks on a piece of white paper. It is an “unthinkable” proposition.



For beings such as ourselves, a contradictory judgment is inconceivable. Its existence need not be considered.

If that be so, how is it that false propositions can exist? It is because words can have a large number of connotations. When the definition of the word is not stated and when the word has a popular meaning, these connotations are not always explicit. Thus two terms of a proposition may be contradictory without the contradiction being readily apparent. It is revealed only when the two meanings which give rise to the contradiction are brought face to face.

For this reason as soon as a judgment has attained a certain degree of complexity, we may never affirm that is free from contradictions. We can never be sure of having explored all the innermost recesses of its words, the number of which recesses we do not know. The only judgments which may be considered as definitely established are those very simple ones whose entire content is immediately apparent.

From all this we may infer the advantages of a method which diverts the current of thought into narrow channels while still permitting us to bring together very simple judgments and derive therefrom a new judgment, much more complex, with the absolute certainty that, if the initial propositions are "true" *i. e.*, free from contradictions, the resulting judgment will be likewise true. This method is that of formal logic which we shall assume to be known. It reduces all reasoning to a single type—

deduction, of which the syllogism is the elementary operation. All the rules of the syllogism rest upon this law *viz.*, that what is true of the group is true of every component thereof. In other words, it rests indirectly upon the Law of Identity.

If two propositions are taken to be true, a third one necessarily results therefrom, and this third one is automatically determined irrespective of ourselves by the application of purely formal rules based upon the position of the terms in each of the two initial propositions. If these propositions are free from contradictions, the resulting one will be also. Of this we are absolutely certain.

Hence we see the enormous advantage of this method. It localizes doubt. It shows us where to look for contradictions. If once the two initial propositions, namely, the premises, are admitted, the conclusion establishes itself, definitely, absolutely, and in spite of ourselves.

Now all reasoning may be stated in the form of a chain of syllogisms, the premises of each succeeding syllogism being the conclusions derived from a certain number of primary syllogisms.

In these primary syllogisms, the premises are either axioms or definitions. Axioms are *a priori* judgments which we are free either to accept or reject. They can form a coherent system only when two axioms having a common term are not contradictory.

A definition is a creative proposition which attaches certain characteristics to a word and by this act gives it its existence. In order for a word to be thinkable, it is necessary and sufficient that no two contradictory characteristics be attributed to it. Once a word has been defined, it contains nothing beyond that which the definition has put into it. Such being the case, a simple examination will demonstrate very definitely whether the particular definition is legitimate.

When a certain system of axioms has been adopted and definitions laid down, results are deduced therefrom in accordance with the laws of our minds.

At this point we shall study in greater detail the characteristics of deductive reasoning, the rules of which constitute formal logic. This study is indispensable to a justification of our conception of scientific theories.

First of all, this logic is called formal because it in no wise penetrates to the depths of the judgments it enunciates. If the premises are true, and this it is unable to tell us, and if certain conditions as to form, based upon the place of the terms in each proposition, have been complied with, then the conclusion establishes itself.

Formal logic is, in short, somewhat analogous to a system of gearing which, when furnished with true propositions, draws therefrom true conclusions. It "presses" the two propositions which are given it just like a lemon and squeezes out a third one to

which the bringing together of the first two has given rise but which had hitherto remained hidden. Before the conclusion had been stated, it was already in existence. It was a reality, but an unknown reality, such as radium was before its discovery. Formal logic merely rendered it explicit. It did not create it. Nothing but a new definition or axiom, incorporated into a new system—*i. e.*, united with a series of propositions already admitted or established, can create new mathematical “ facts ” which then need only to be rendered “ explicit.”

It is important to insist upon the mechanical nature of the reasoning machine. We are at liberty to make it press out this or that portion of the initial proposition, but, once we have chosen the way in which it is to function, we are no longer its masters. It proceeds alone and wholly independent of us, in a manner absolutely determined, and without any measure of freedom. Thus, in order to modify the conclusions to which it leads, we have only one means at our disposal, namely, to modify the material which we feed into it to grind, or in other words, to change the original propositions, *i. e.*, the axioms or definitions.

While it is true that initial identical propositions necessarily give identical conclusions, the reverse is not necessarily true. Identical conclusions may be made to follow from different premises.

Finally, the formal character of the reasoning machine imposes a restriction on its use. Since its rules

are based upon the place of words in propositions, it can operate on propositions only. A thought, whatever it may be, cannot serve as the point of departure for deductive reasoning unless it has first been translated by words into the form of a proposition. The reasoning machine cannot begin with ideas, notions or vague representations. The only intermediary between it and our ideas is the initial propositions which "hook up" the machine with our thought.

This is important for the purpose of justifying the conception of the physical sciences which we shall develop further on and also for judging the practical worth of formal logic and the nature of the certainty which it gives us.

We saw above that, if we accept the initial propositions as true, the consequences which are logically deduced therefrom, *i. e.*, in conformity with the laws of our minds, must be likewise accepted. But we do not know whether the laws of our minds are those of the external world. Nothing proves to us that in nature phenomena are linked together just as the reasoning machine links together in our minds the expressions of these phenomena which we have given them. Nothing proves to us that our logic is that of things. Furthermore, does the expression we give to a phenomenon, in order that we may be able to subject it to the rules of formal logic, represent that phenomenon in its entirety and does it represent nothing else? We do not know. So even if we were

certain that phenomena are associated in the world just as propositions are in a syllogism, we should still be unable to affirm after having passed several rungs (of the logical ladder) that our deductions were still representative of sensible reality.

Thus, to these two questions, namely, the validity of our logic for the external world, and the exact representation of phenomena by the propositions which translate them, the classical conception of physical reality does not permit of a reply. We shall see further on that these questions do not come up and are, in fact, meaningless.

Is the deductive reasoning which we have just studied the only implement used in building up the sciences? Certainly not. Beside it and along with it, we employ reasoning by induction and we use the infinite power of intuition.

In reasoning by induction we must distinguish between two forms which are entirely different: mathematical inductive reasoning and physical inductive reasoning.

The first is based upon the "affirmation of the power of the mind which knows itself to be capable of conceiving the indefinite repetition of an act, provided that such act is possible once."<sup>1</sup> It simply implies the self-consistency of the mind, of which we have direct intuition.

The basis of physical induction is of quite a different nature. It is the belief in the identity of nature

<sup>1</sup> H. Poincare—Science and Hypothesis.



with herself, the affirmation that a relation observed several times is fixed and eternal. It is, in short, the projection into nature of one of the laws of our minds. The basis of physical induction is expressed in the Law of Causality, which, in the classical conception of science, can be only a simple hypothesis. Further on we shall give a different conception of this.

What it is important to note is that physical induction and intuition are procedures of research, not of proof, and that they have no place in a science which has already been built up. Let us open a book on geometry, mechanics or physical theory. We find therein a series of deductions proceeding from definitions laid down *a priori*; *e. g.*, forces, masses, atoms, energy, probability. Nowhere do we see explicit reference made to the belief in the existence of laws in nature or to intuitions, conscious or otherwise. The discussion which follows will state this precisely and will prove such statement.

To sum up, there are only two mechanisms in the reasoning machine—deductive reasoning properly so called and inductive mathematical reasoning. The latter may be reduced to syllogistic reasoning which is thus the fundamental operation of formal logic.

There remains to examine more closely one of the forms of the reasoning machine, mathematical analysis, which, by reason of the rôle it plays in the construction of the sciences, deserves particular study. Mathematical analysis is a specialized reasoning ma-

chine, one which can be set in motion only with a special category of propositions, *i. e.*, those which represent magnitudes each of which is condensed into the form of a symbol through the intermediary of a definition. On these grounds it is, as Leibniz said, only a special elaboration of general logic.

The premises of analytical reasoning are axioms or definitions. Axioms are very general propositions which seem to express laws of our minds. We shall see later what reasons controlled their choice. Definitions attach to a symbol a certain number of characteristics and permit the application of the rules of syllogistic reasoning to this symbol by expressing in words the properties which were included therein.

We are then masters over the definitions. We are at perfect liberty to create *a priori* definitions. A single condition limits our creative power. It is that we must never violate the Law of Identity within one definition or within two definitions having a common term.

With this principle once laid down we can see that mathematical analysis is a method which permits us to write the result of complex intellectual operations by the application of purely formal rules to the initial symbols. We could obtain this result directly by applying the rules of formal logic to the definitions of the original symbols. This would be long and inconvenient. Instead of this, mathematical analysis goes through the operation once and for all under as general a form as possible. It keeps the result and



enunciates a rule which permits us to recover it in each particular case, mechanically and surely, without going into the basis of the operation gone through.

A single instance will show us the characteristics of this method. I wish to solve the system of simultaneous equations:

$$3x + 2y = 5$$

$$4x + 3y = 4$$

I can solve it directly in this particular case. The solution is immediate and is furnished by ordinary common sense. I multiply the two members of the first equation by 3, those of the second by 2, and subtract member from member. I thus obtain the value of the unknown,  $x$ . This operation is simple because there are only two unknowns. Its complexity increases rapidly with the number of unknowns and becomes practically unworkable when this number is large.

Mathematical analysis spares me the trouble of going through the operation. It has done it once and for all and gives me, therefore, a purely formal rule for the association of symbols, which, thanks to the theory of determinants, gives me the values of the unknowns sought. The value of each unknown is a fraction whose denominator is the determinant of the coefficients of the unknowns and whose numerator is derived from the denominator by replacing the coefficients of the unknown under consideration with the independent terms, their signs being changed.

The theory of determinants is thus quite characteristic of the methods of mathematical analysis since it brings out the manner in which the latter saves us from reasoning by replacing complex intellectual operations with rules of writing.

For this reason mathematical analysis is just like formal logic, which, having once and for all made certain of the legitimacy of the rules which it states regarding the association of propositions, gives a mechanical method of writing the proposition resulting from any two given original propositions. Thus one can see in mathematical analysis all the characteristics of formal logic. We shall not again take up their enumeration.

The parallelism existing between ordinary geometry and analytical geometry illustrates still further the profound identity uniting those two mechanisms. Geometry is the application of formal logic to mathematical entities, straight lines or planes, for example, defined by propositions. Analytical geometry is the application of mathematical analysis to these same mathematical entities represented by symbols, reasoning having demonstrated the equivalence of the two representations.

The parallelism exists because the implement is the same, although it is fitted up differently in order to utilize in each case equivalent materials furnished in different forms. This parallelism is further proved in a very characteristic manner by the existence of modern geometry, in which the symbols

of algebra are subjected to the rules of formal logic by our giving them the names and definitions of the corresponding geometrical entities in order to make them fit into the reasoning. At any moment it is possible to pass from one form to the other as a consequence of the absolute equivalence of the two procedures of transformation, analysis and logic, to which we reduce the reasoning machine.

## PART TWO

### THE SO-CALLED PHYSICAL SCIENCES

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#### CHAPTER V

##### GENERAL CONSIDERATIONS

We in nowise distinguish the mathematical sciences from the physical or natural, nor from the social sciences. We claim that all sciences are of the same type, each comprising an experimental or observational branch which gleans the facts and extracts therefrom empirical laws, and a rational branch which “creates causes.”

We shall sketch the outline of some one of these sciences. The following pages will demonstrate the generality of this outline.

The point of departure is living man grappling with this something which resists him, which he calls reality and which reveals itself to him only in a succession of sensations. All that is real, all that is given to him, is this series of sensations and nothing else. By observing, by experimenting and by living, he derives from them general rules which are the expression of the common character of a certain group of sensations and which serve to direct his future action. But these rules are no more a reality than is the sphericity of marbles. Before man had asserted them they did not exist. We shall see that

they cannot exist without him and that he creates them to a large extent.<sup>1</sup>

Now, human reason is governed by the laws of Identity and Causality. It wants a "nature of things" to exist, to be made up of things identical with themselves, and to be the causes of observed phenomena. Since these things, such as molecules, for instance, are not furnished to the mind by the senses, it creates them by condensing a certain number of characteristics into the words which represent them. This *creation of causes* is the fundamental part of theoretical science.

These causes were not chosen at random. They are propositions, axioms, and definitions so chosen that the immediate conclusions drawn by the reasoning machine from them will coincide with the expression of the laws already enunciated by the empirical branch of science at the moment of their creation. Therefore, the coincidence need not surprise us. The premises were chosen with the view to establishing it.

The premises once posited, the reasoning machine does not hesitate. From their juxtaposition, it derives an ensemble of propositions which constitute rational science or science properly so called.

These propositions are supposed to express the laws of the sensible world. If these laws are already known, the theory is thereby confirmed. If they are not yet known, the scientist is supposed to utilize observation and experimentation in order to dis-

<sup>1</sup> Ch. VIII Mechanics.

cover them in nature. If he does not succeed in this, the causes cease to be true. They will have to be either altered or replaced.

We must note, furthermore, that the road, which the reasoning machine follows and which results from the arbitrary choice of the propositions juxtaposed, will, in general, be that toward which attention has been drawn by the experimental discoveries of the moment.

The important thing in this exposition to remember is the fact that, at the beginning of every science, the empirical branch is necessarily anterior to the rational. The latter is a physical theory whose foundations were laid only that they might enable us to rediscover by deduction the laws which the former had asserted.

A separate study of each of the sciences will confirm this viewpoint.

## CHAPTER VI

### RELIGIONS

The first system revealing to us at the outset the "nature of things" was mythology. All phenomena were thought to be willed by beings more powerful than ourselves but similar to us. Their wills were the causes of observed appearance. Thus was the first physical theory constructed.

It is quite easy to imagine how man, thrust into the midst of life, was led to discover the gods. The first cause for wonder on the part of a child is to find himself the cause of a phenomenon. His first amusement is to exercise his power over things. Observing the marvellous results of his activities, he becomes cognizant of the manner in which he brings them about. He pushes the marble and the marble moves. Therefore, he is the cause of its movement. But later on, when he becomes a man, he perceives the limits of his power. He is unable to make rain and he cannot make the billows surge, yet the rain falls and the billows surge. The explanation is quite simple. Somewhere there must exist beings with powers more extensive than his who have "willed" the rain or the storm. Since they have willed it, they, like himself, must be endowed with a will, but a will much more powerful than his own. These are the gods who have names, faces, feelings, and a past. The causes are

created, the phenomena explained, and reason is satisfied because it has attributed to self-consistent beings a quality, *i. e.*, omnipotence, which explains observed phenomena. The disturbing whys are no longer left unanswered. There is now a “ physical theory ” which explains everything.

We shall see further on <sup>1</sup> how it became necessary to modify this physical theory in order to adapt it to newly observed appearances. Man was brought to lend to the gods, which he created, a moral character just as the definition of the atom was made more and more complex in order that it might explain newly discovered phenomena.

<sup>1</sup> See Ch. XIV. Theological Morals.



## CHAPTER VII

### GEOMETRIES

At the base of geometry, as of the other sciences, there are rules of action. A man has a field and, when he wishes to divide it in order to endow each of his two sons, he invents land-surveying, an aggregate of rough rules for dividing a piece of land into parts which will produce equal quantities of grain. The empirical branch of geometry would thus be the science of the surveyor.

History seems to confirm this point of view. There is extant an Egyptian work, the manual of Ahmes<sup>1</sup> which is thought to have been written between 2000 B. C. and 1700 B. C. This lays down an aggregate of practical rules designed to solve concrete problems: "rule for measuring a round fruitery; rule for laying out a field; computation of the fodder consumed by geese and oxen."

"The author knows just enough arithmetic to meet the requirements of reckoning. Speculative and disinterested sciences are unknown to him."<sup>2</sup>

<sup>1</sup> This manual is cited by M. P. Boutroux in his "*Principes de l'Analyse mathématique*" (Hermann, publisher). It has been published and translated from the Papyrus Rhind of the British Museum by Eisenlohr (*Ein mathematisches Handbuch der alten Aegypten*, Leipzig, 1877).

<sup>2</sup> Pierre Boutroux.

It is, furthermore, important to note that the rules which he lays down are materially different from the theorems of Euclidean geometry and prove that he was ignorant of those theorems, his rules being only very rough approximations of them. Thus, the area of a triangle of which the angle A is small is one-half the products of the sides AB and BC. The practical rules of surveying, or what might be called the empirical laws of space, would accordingly seem to antedate the construction of rational geometry, of which "Euclid's Elements" furnishes us the type.<sup>1</sup>

To imagine the processes by which these practical rules were discovered is to review the entire history of the human mind. But it is not possible to reconstruct that history. It is not a regular, well-ordered structure which can be depicted in words. It is a continuous process with neither beginning nor end, a progression in which observation is inherent in each and every second, experience in each and every minute, in which all work takes place in a half-consciousness, that of children and of primitive men. It is a vague intuition, trials repeated indefinitely, in which success helps life and failure kills it. It is the product, not of a systematic study of the external world by the mind—in primitive life there is neither

<sup>1</sup> Observation of primitive societies seems to confirm this point of view. According to M. Léon Brunschvig uncivilized peoples indulge in barter and operations involving calculations without having any idea of the rules of equality, or of the laws of addition and multiplication which give their acts a character of truth. (*The Halting-Places of Mathematical Philosophy*, p. 5.)

external world nor mind—but the product of the only reality known to us: ourselves, our life flowing on with its sensations and its appetites. Rules are formed along with ourselves, little by little, just as our personality and reason are. Collected by Ahmes, they constitute the “ Rules for measuring a field.”

These rules once laid down, it becomes necessary to explain them, to find, or rather, to create their causes. The first geometricians, guided by intuition and by innumerable inductions and analogies among the objects revealed to them by their senses, proceeded to invent a system of propositions, axioms, and definitions capable of being the cause of the empirical rules laid down.

For this it suffices that the reasoning machine, operating on these propositions, can draw therefrom conclusions whose expression coincides with the rules of surveying. However, once set in motion, the machine goes much further than the end aimed at. It is possible to continue indefinitely bringing together the newly obtained conclusions and the axioms or conclusions previously obtained. We thus derive the infinite succession of theorems of geometry.

The initial propositions were divided into several groups. Some very general propositions, which seemed to express the laws of our minds, became the axioms. The others, which created “ mathematical entities ” seeming to approximate the objects revealed to us by our senses, became definitions. These condensed into a single word, which named what was

defined, the sum-total of all the characteristics enumerated by the definition.<sup>1</sup>

The mathematical entities thus created are not given to us by our senses. To confirm this we need but recall what was stated above concerning the characteristics of formal logic. Our senses can merely furnish us images which can in no wise serve as bases for deductive reasoning. Such a base can be only a proposition creating what is defined by ascribing to it a certain number of characteristics. All we can affirm in this process is that, when we attempt to discover in reality a physical thing possessing roughly the characteristics of the defined entity, it will resemble certain objects revealed to us by our senses. But the statement that the definition expresses the essence of an object of the external world does not and cannot have any sense. An object is a sum-total of sensations. A definition, on the other hand, is a sum-total of non-contradictory words. The two are of distinctly separate orders.

The mathematical entities created by definitions constitute geometrical reality, Euclidean space, "our" space since it is the space of the surveyor.

The existence of three-dimensional Euclidean space is thus a physical theory permitting us to re-

<sup>1</sup> When the definition of a geometrical entity designated by one word is not explicitly given, it is determined by the axioms of which the word is a term. Thus, when in certain treatises a straight line is not defined, the axiom which states that through any two points in space a straight line can be drawn and that no other straight line but that one can be so drawn, is nothing more than a disguised definition.

discover, by applying the reasoning process to suitably chosen propositions, rules empirically discovered. It is in no wise imposed upon us in its precise form by an anterior reality which it describes exactly. It was chosen as the "true" space because it accounted perfectly for the empirical relations adopted as the laws of space, *i. e.*, because the system of consequences which it was possible to derive from it coincided with the empirical branch of geometry.

Is this coincidence complete? Is three-dimensional Euclidean geometry a perfect physical theory? Recent developments of mathematical physics seem to prove the contrary. The non-generalized theory of relativity forces the consideration of a four-dimensional Euclidean "universe." If the perceptions of our ancestors had been sufficiently keen to observe the relations connecting distance with speed—and this is not inconceivable—they would have been brought to construct a four-dimensional Euclidean geometry rendering it possible to translate experimentally determined relations into the language of analytical geometry. Like all physical theories, the classical geometry of three dimensions interprets the results obtained up to a certain stage in the advancement of the science. Beyond that point it ceases to account for the appearances observed. It must be either altered or replaced.

This conception of geometry as a physical theory comes into conflict with a universal feeling regarding space as one of the conditions of intelligibility, *i. e.*,

one of the forms of the human mind. Four dimensional space is regarded as impossible because inconceivable. This objection seems untenable. It is our education which causes us to develop in "our" space the system of our sensations. A child wishing to seize a distant object does not seem to take account of the existence of three dimensions to any great extent. Furthermore, language has become for us one of the conditions of thought. Conscious thought does not exist without words and yet we think in French while others think in Russian. To each people, the words they employ seem to be a form of their minds. Mathematical analysis itself seems to us fundamental and constitutive of our reason. Now integral calculus only dates from the Seventeenth Century. The perfect assimilation of these ideas which have become the very condition of our thought is a result of education. We do not believe it impossible for a mind suitably educated to conceive of a sensible space differing from ours.

The confirmation of the fact, that real space is nothing more than a coherent system of definitions making the interpretations of observed appearances possible, is found in the existence of non-Euclidean geometries. These are geometries whose mathematical entities are created by propositions neither self-contradictory nor contradictory to one another, but in which one of the initial propositions, an axiom or definition, is contradictory to one of the initial propositions of Euclidean geometry. Considered apart



and in themselves, they present no contradictions. From the rational point of view they are perfect.

It is possible to demonstrate that beings, endowed with our reason but evolving in a world different from ours, would have been led to adopt as their true space a non-Euclidean space, and, inversely, that beings living in our world with a rational make-up other than our own would have constructed a non-Euclidean geometry.

A demonstration of the former proposition was given by Henri Poincaré. He imagines a world inclosed in a great sphere and subject to the following laws: the temperature is greatest at the center, diminishing as you go away from it and reaching the absolute zero at the outer extremity of any radius. If  $R$  is the radius of this sphere and  $r$  the distance from the point under consideration to the center of the sphere, the absolute temperature is proportionate to  $R^2 - r^2$ . All bodies have the same coefficient of expansion and an object moved from one point to another immediately attains calorific equilibrium with the environment. Finally, the index of refraction is inversely proportionate to  $R - r$ . Nothing in these hypotheses is contradictory or unimaginable.

Now, though that world be finite for our geometry, it is infinite for its inhabitants since, as they approach the outer limits of their sphere, they cool, becoming smaller and smaller, their steps therefore becoming shorter and shorter, and they are thus never able to reach the outer limit of their space.



Geometry in that world would consist of a study of the laws governing the motion of solids deformed by variations in the prevailing temperature. The space which the inhabitants would designate as real would, therefore, be non-Euclidean. This shows sufficiently that that space would have been created by the beings we have imagined for the sole purpose of interpreting observed appearances in the world in which they lived.

On the other hand, beings living in our world and observing the same appearances as ourselves, but with a rational ego having laws other than those of Identity and Causality would have constructed a geometry different from ours.

As a matter of fact, the appearances which we observe in our world are all determined by our way of measuring distances. If we measured them differently, the appearances observed would likewise be different and our geometry would no longer be Euclidean.

Let us suppose, for example, that we had defined the unit of length as that distance which a body chosen once for all would traverse in the void in a second of our time. This unit, measured by our meter would decrease inversely as the square of the altitude. In the system adopted, this unit would be constant by definition and bodies which we call solids would be deformed by changes in altitude. The geometry which we, armed with such a unit of measure,

would construct would obviously be different from ours. Space would no longer be Euclidean.

Why did we not adopt such a unit? Why does the supposition that we might have done so appear absurd? It is because our sensations, viewed through the medium of such a system of measuring, would no longer obey the Law of Causality. Its temperature remaining the same, the length of a metallic bar would vary with the altitude, whereas intuitively it seems to us that altitude does not enter into the phenomenon in question. The law of Causality would no longer be satisfied.<sup>1</sup>

This latter example shows us to what extent the human mind enters into the creation of appearances presented to us by the sensible world. It caps the justification of the sentence written above, "Empirical laws cannot exist apart from ourselves—they are largely created by us," (Chap. V). Chapter VIII will develop this statement further.

The preceding considerations reveal to us the nature of our geometry. It is the best physical theory at a given stage of scientific advance for beings having our minds and evolving in our world.

The development of mathematical physics confirms this point of view. We said that the ungeneralized theory of relativity expressed itself in a Euclidean space of four dimensions. The generalized theory of relativity compels us to adopt a non-Eucli-

<sup>1</sup> We shall take up again in Chapter VIII a detailed demonstration of this theory, which we owe to M. Painlevé.

dean space in which the phenomena we observe display themselves as "our" space. This makes it seem necessary wholly to abandon the conception which represents space as an *a priori* form of sense perception.

Regarding geometry as a physical theory gives prime importance to the initial propositions, axioms and definitions. It is these propositions which determine the form of the whole structure and which fix with absolute finality the content of the theorems of which it is composed.

In order to vary the enunciation of these theorems with a view to making them conform to empirically discovered laws, we have but one means at our disposal, *viz.*, modifying the initial propositions. Now, though the definitions are clearly expressed in a great number of treatises, the axioms are for the most part implicitly assumed. The work of rendering them explicit is extremely delicate. It has been done by M. Hilbert who bases his geometry upon twenty-one axioms divided into five groups.

These axioms once stated, the entire edifice may be constructed merely by employing the resources of formal logic. It is, furthermore, possible to construct partial geometries by using only a certain number of the stated axioms. We thus see which of the propositions survive and the axioms that each of them imply.

Because of the prime importance of the geometrical method of proof in building up a rational system

of morals, we set out the rules which, according to Pasch, should serve for the foundation of a geometry:<sup>1</sup>

1. "We should state explicitly the primitive concepts by means of which we intend to define all the others (definitions) ;

2. "We should state explicitly the fundamental propositions (axioms) thanks to which we propose logically to derive other propositions (theorems) ;

"These fundamental propositions should appear as pure logical relations among the primitive concepts and should be independent of the concrete meaning which we give to these concepts. They should be compatible and, so far as possible, independent."

These fundamental rules will become for us, as we shall see in the following chapters, the rules which ought to govern in the exposition of a science of any sort.

<sup>1</sup> Cited by M. P. Bontroux—The Principles of Analysis.

## CHAPTER VIII

### RATIONAL MECHANICS AND CELESTIAL MECHANICS

Mechanics, or the science of motions, has for us a particular interest because of the manner in which it is organized. Innumerable appearances were given us but no means of interpreting them. We shall see here, perhaps more clearly than in the other sciences, that the meaning of these appearances might have been considerably modified by a change in the conventions which we used to express them.

In order to treat mechanical phenomena with some precision, it was necessary to determine at the outset the order of their succession. It was necessary to place them in time whereas they occurred for us only in a personal and subjective duration. It was also necessary to measure distances and to mark movements in space, that is, to define the axes to which they might be referred.

Now, no way of measuring time or distance, and no absolute axes were given. Very little reflection will enable one to see that upon the choice of these methods the empirical laws of mechanics would depend. The falling of bodies is not uniformly accelerated unless our clocks have a constant movement. If they increased in speed with reference to the celestial sphere, bodies would no longer fall in accordance with the same law. Similarly the Law of Inertia is

only to be verified by reference to axes suitably chosen. This shows clearly that the rôle of the human mind is not purely passive in the observation of the laws constituting the empirical branch of science. Man does more than merely record appearances. He determines them to a great extent. There is something which exists outside of him but he can only give expression to it after he has added to it much of himself. To say that the empirical laws of our world have existence in themselves is an affirmation without any meaning. They exist only when they have been stated and they might have been different. Underneath the propositions which state them, there is something which resists us but its form is completely unknown to us.

With this laid down, let us see how the empirical branch of mechanics might be constructed. The whole difficulty comes from the fact that the choice of the fundamental definitions—distance, time, and absolute axes—determines the laws of mechanics, whereas these definitions can only be chosen by reference to the form which it is proposed to give to those laws. It is the affirmation that the rotation of the celestial sphere is uniform which fixes the unit of time. The uniformity of this rotation cannot, therefore, be an experimental fact. It is merely a definition.

The reasons why we adopted the definitions which are at present the basis of mechanics are formulated in a masterly manner by M. Painlevé in his course in



the Polytechnic School. We are afraid that we should disfigure them by not quoting them verbatim.

“Let us admit for an instant that it is sufficient to adopt once for all a material rod to measure distances and some clock to measure time, without troubling ourselves to find out whether the length of the rod remains constant or whether the clock is well regulated. It appears immediately that such a method is inadmissible. In fact, our experiences verify the Law of Causality only when we measure distances and duration in the classic way, *i. e.*, in meters and in sidereal seconds.

“On the other hand, if we replaced our sidereal clock by one which was constantly accelerating with respect to the first, the Law of Causality would cease to be verified. A chemical reaction, which had an invariable duration, when repeated under the same conditions and under our first system of measurement, would, with our new measures, seem to be slower at each new trial. Analogous remarks apply to the measurement of distances.”

Thus we defined the unit of time and of distance which give expression to phenomena and which seem to verify the Law of Causality. They are: the meter or one forty-millionth of the terrestrial meridian; the second or  $\frac{1}{86,400}$  of the sidereal day. Then we had to say that the same rod kept under conditions seemingly rigorously constant would have, at two periods sufficiently removed one from the other, different lengths if they were expressed by means of the meter thus defined. The Law of Causality seemed to be at fault since identical antecedents produced different appearances for the length of the rod in the two measurements. In order to avoid this, it was

found necessary to define the standard meter as the length of a ruler kept under certain conditions and to admit that the length of the terrestrial meridian expressed by means of this ruler was susceptible of variations. We were then able to find in existing physical theories causes explaining these variations which did not invalidate the Law of Causality.

We are, for the same reasons, today considering replacing the standard platinum meter by the wave length of a certain radiation and replacing the sidereal second by a unit of time derived from the measurement of electrical resistances.

Thus the practical units of measurement are essentially provisional. They can only be considered as those which approach nearest to ideal units defined *a priori* as giving to phenomena expressions verifying the Law of Causality. That law, then, does not govern sensible appearances unless distance and time are suitably measured. It is not something given. We have imposed it upon the world; but, what was not evident, the imposition of it was possible.

We must remember carefully all the preceding considerations. They will permit us in Chapter XV to define the ideal unit of value in exchange and to explain the choice of our practical unit, the monetary standard.

In mechanics, furthermore, considerations of a similar order determine the axes to which we refer all motions. It is not possible, in fact, to define the trihedron of reference otherwise than by stating that



it is the one by reference to which the axioms of our mechanics are verified. Hence, the fundamental enumeration of the bases of mechanics which we borrow once more from M. Painlevé:

“It is possible to adopt once for all measures of distance and time, and a trihedron of reference such that, in addition to the axioms of Euclidean geometry, the following laws will always be verified:

Axiom I: Law of Inertia.

Axiom II: Law of Action and of Reaction.

Axiom III: Law of Initial Conditions.

Axiom IV: Law of the Parallelogram.”

Thus, it is impossible to define separately distance, time, the trihedron of reference, or the axioms of mechanics. These definitions are necessarily simultaneous. They exist only by virtue of each other and have been chosen only in order to satisfy certain laws of our mind. They form a synthesis in which our contribution equals that of the outside world, an aggregate which we were able to create only by countless gropings in a darkness where nothing was given and where everything was swayed by everything else. Each point could be fixed only provisionally since it modified all the others. The history of the tentative efforts which gave the present coherent form to our system of definitions best reveals to us the true nature of our sciences. This groping is going on under our eyes. The mechanics of relativity is thus trying to create itself. All the difficulties it encounters reside in the definition of its bases, in the fact of their

interdependence. Taken separately, they have no sense. Only in their synthesis can they be defined.

The empirical branch of mechanics being thus constituted, rational mechanics presents with great clearness the characteristics of a physical theory. The empirical rules to be interpreted are those revealed to us by celestial mechanics and by physics, rules expressed by means of units whose utilization came about by an historical evolution, by a vague intuition of the reasons which we have just explained. These rules are the observations of Tycho Brahé or the laws of Kepler resulting therefrom, the laws of falling bodies, the laws of the pendulum. These laws once ascertained, it remained, following out the general formula, to create their causes.

Now when we move an object, we know the cause of its movement, namely, the effort which we have applied to it. Yet our senses reveal to us the existence of bodies which are displaced without the intervention of any human influence. We can perceive no cause for their movement. As usual, we imagine this cause by creating forces similar to our effort and making the movement of an object in all cases merely a manifestation of the effort exerted upon it.

The notion of a physical thing capable of causing motion being thus created, we are obliged to give a definition of it which can serve as a base for a rational theory. This definition is one that gives for each movement the magnitude of the force whose ex-

istence must be imagined if we would explain the movement observed; it is the expression

$$F = m\gamma$$

This definition taken in itself already informs us of the nature of our explanations. Force, according to our theories, is the determining cause of the movements we observe. It is, in a way, an invitation to movement, which should precede movement as cause precedes effect. Now force is defined only by knowledge of the movement whose appearances we seek to rediscover. We thus see that it is only an *a priori* explanation, a logical cause which is created in order to explain the appearance of a phenomenon, but which can only be determined by knowledge of the phenomenon to be explained.

In any case, this definition, joined to the axiom of universal attraction and to the axioms of mechanics previously stated, permits us to construct by the deductive method a marvelous edifice all of whose theorems coincide, in proportion to the accuracy of our observations, with rules empirically determined. Therefore the created causes, axioms and definitions, are satisfactory. Physical theory is at present true for us, granted the apparatus which we employ.

Will it always be so? We do not think so. Here again the theory of relativity seems to indicate that the causes will have to be modified in order to explain appearances not yet interpreted.

It is readily seen how far we are from space and time as something imposed on our understanding, as

*a priori* forms, or as necessary frameworks for our thought. Like the laws of inertia or of universal attraction, like the sphericity of the earth and its movement around the sun, space, time, and forces are all devices invented by us in order to render intelligible the aggregate of our sensations.

We shall see in the following pages that this is characteristic of all of our knowledge of the social world as well as of our knowledge of the physical world.

## CHAPTER IX

### PHYSICS AND CHEMISTRY

We have been led to assign to the human mind a rôle of considerable importance in the interpretation of the phenomena of experience. In consequence, the historical order in which the study of the various sciences was undertaken must have determined their form to a great extent. Now physics was not systematized until after astronomy had definitely fixed the unit of time and the axioms of our mechanics. These axioms and our way of measuring time were, until modern times, considered as having been definitely settled and their validity was not questioned. We were, therefore, able to observe nature only through the medium of these conventions, which fact determined to a great extent the empirical laws of physics and limited considerably the number of possible theories.

1 We shall not take time with the *empirical* branch of physics. The basis once admitted, it is reduced to a study of our sensations or, more exactly, of our perceptions, which are our sensations seen through the prism of previous conventions, whether explicit or not. This branch of science discovers and enunciates laws.

The *rational* branch is devoted to the construction of physical theories, that is, once again, a system of

axioms and definitions which when “squeezed” by the reasoning machine will yield conclusions coinciding with observed appearances. It is again, and here more than ever, the creation of causes.

As for the method which permits the creation of axioms, such as the law of the conservation of energy or those of probabilities, and the creation of physical entities, such as potential energy or the molecules of the kinetic theory, that method is always one of intuition and vague induction. Its progress is guided at each step by the boundaries of deductive reasoning, which test the coincidence of the results obtained with the phenomena of experience.

This conception of physical theories is confirmed by our adaptation of causes. When a new phenomenon is observed, several cases may present themselves:

Either a proposition explaining the observed phenomenon has already been drawn by deductive reasoning from the initial system of propositions adopted, or else it is possible to derive one therefrom. In either case the theory becomes “truer” than it had been before. This being so, the created causes should be left unchanged.

On the other hand, this operation may be impossible. In that case we try to adapt the causes, *i. e.*, to add to one of the definitions a characteristic which, incorporated into the aggregate of initial propositions, latently contains the expression of the observed phenomena thereafter needing only to be rendered explicit by the reasoning machine.

If we are not successful in doing this, it becomes necessary to upset the edifice and to replace its bases by new foundations. The undulatory theory replaced the emission theory when the latter failed to explain the empirical law, which Foucault had discovered, showing the ratio existing between the indices of refraction and the inverse of the speed of light. The example is too classic to need developing. It shows clearly how empirical law creates theory. This fact, which now appears evident, will have to be kept in mind when we take up the study of social theories.

We must, however, answer an objection which will certainly present itself to the mind of the scientist: "When one studies chemistry," says he, "it is impossible to escape the feeling that we must affirm with certainty the existence of molecules, a certainty resulting from quantitative predictions later verified which are too striking to be as satisfactorily explained by any different cause, a certainty which will be indisputable the day that we shall have observed molecules visually."

This point of view so potent in the environment of the laboratory cannot withstand the criticism of the philosopher. Even if we had seen molecules, we should still contend that the bases of the atomic theory had not been imposed upon us but created by us, and this because the image of a molecule can in no wise serve as a basis for deductive reasoning. Moreover, we need only recall what we said regarding the necessarily verbal character of premises in order to



recognize the fact that the molecule, which is at the basis of the kinetic theory, is not one which the microscope can reveal to us, but is a sum-total of characteristics imagined for the exactly defined purpose of accounting for the observed properties of gases. The molecule of the kinetic theory is a definition, not an image.

In any case, if the definition ascribed to the thing defined a characteristic visually observable with the means at our disposal, this characteristic would have to be observed or the theory would be false. Thus the theory of gravitation is now perfectly true because, after the calculations of Leverrier, the planet Neptune, a necessarily visual support of the attractive force imagined, showed itself through our telescopes to be in the place where it should be. The kinetic theory is true, because the Brownian movements reveal themselves to our eyes. If this had not been observed, the kinetic theory would have been only a little less true. It would have sufficed in that case for us to have assumed that the molecular agitation was not transmitted to particles as large as the smallest which we were able to observe. Furthermore, if we had arrived at the degree of skill necessary for molecules to show themselves through our microscopes and still did not see them, it would have sufficed to have imagined the molecule to be an electro-magnetic disturbance or something of the same nature, and the cause modified in this way would have continued to interpret the negative phenomena discovered.



In order to ascribe to sensations their true place, we must remember the considerable elaboration that our brute sensations undergo before they become images. We must remember that they cannot be translated into propositions unless they are seen through the system of axioms already adopted, that we do not judge from the coincidence of propositions with sensations but from the coincidence of a deduced proposition with another proposition which translates the sensation, and, finally, that this translation is made, and can only be made, with our language and our analysis. Thus it will be understood why we refused to admit physical reality to be the exact and brute reproduction of the external world.

The kinetic theory, for example, is based upon the axioms of the calculus of probabilities; the physical entities which give it its existence are the molecule and the most probable state. The theory of the conservation of energy is rendered possible only by positing a potential energy which is nothing but a number defined by that theory's methods of measuring. A statement that these notions are purely and simply derived from the aggregate of our sensations would be completely devoid of meaning. Here, as always, we have a system of propositions created by our minds in order to be, like the theorems of geometry and those of mechanics, the premises for reasoning whose conclusions coincide with the propositions expressing the properties empirically discovered.

It now suffices to recall what we said about deductive reasoning in order to deduce therefrom certain characteristics of the causes thus created. The reasoning machine, as we saw, is an automatic instrument whose operation is absolutely determined by the choice of initial propositions. It follows from this that the only means at our disposal enabling us to fit the conclusions to observed appearances is to modify those propositions. At that point alone may the machine be controlled. The rest is inaccessible to us.

The initial propositions, axioms, and definitions are thus the only intermediary between reality and our science. They form the bridge between the rational edifice which we attempt to construct and the unknown world in which we live. They are the causes of the appearances observed just as the nature of any hypothetical thing whose existence we might affirm would be. For this reason they are the only physical part of science.

We are now able to answer an objection raised in Chapter IV. Formal logic, we said, is a method of deduction in accordance with the laws of our minds, but we do not know whether our logic is that of things, *i. e.*, whether phenomena associate themselves in nature as the reasoning machine associates in our minds the expressions that we have given to them.

When we consider the admirable fruitfulness of analysis as applied to physics, we are tempted to

reply in the affirmative—logic does rule the world. We maintain that this statement does not and cannot have any meaning.

Let us observe first of all that nature, in the present state of our methods of observation, reveals the existence of a strict determination and in no wise the existence of a causation. Phenomena succeed each other in an immutable order. In order to be able to affirm that this order is the logical order, we should have to know the nature of things, the essence of the objects that constitute the world. Now we have seen that there is no method at our disposal for discovering this. Our knowledge is built up only on sensations which are nothing other than an aggregate of appearances.

But there is more to be said. The notion of cause is essentially subjective. It could not exist outside of a mind conceiving it. We believe we have shown that the Law of Causality is not imposed upon us by the world, but that it is perhaps for us the most convenient method of adapting ourselves to the world. We have shown that in no case did we have the causes given, but that they were always created by us for the sole purpose of enabling us to find rationality in the succession of observed phenomena. Thus the coincidence of our theorems with those phenomena is not a sign of their reality as causes. They were created for the sole purpose of making that coincidence certain. They are modified or done away with when such coincidence is no longer pres-

ent. Thus, even the fact that the imagined causes have enabled us to foretell phenomena does not prove that there are causes in nature. If the logical forecasting of a new phenomenon had not been possible, and this has often happened, the causes would have been abandoned or changed.

Thus logic and mathematical analysis are pre-existent to our science. In no case could the latter modify their laws. It can expedite their development along certain lines but it cannot change their nature. What confirms this point of view is that the methods of integral calculus, for example, have never been questioned. The idea that they could have been adapted to the nature of things appears absurd to us—yet their discovery dates only from the seventeenth century. Similarly, when we were unable to deduce from the theory of emission the results of Foucault's experience, no one thought of questioning the deductive method.

The laws of physics, those of space or those of gases, are all simultaneous. If we establish a hierarchy among them, some resulting from others by deductive logic, it is in order to permit our minds to understand them. And it was in order to understand them that our minds created causes out of which we have made the external world.

## CHAPTER X

### THE NATURAL SCIENCES, BIOLOGY

We could repeat in regard to the biological sciences all of the preceding considerations. Suffice it to state that they apply in their minutest detail except that, in many of these sciences, the empirical branch which collects appearances is much more developed than the rational branch which seeks to explain them.

The theory of evolution, for example, is a physical theory just as much as that of the conservation of energy. It amounts to an axiom which permits the reasoning machine to interpret rationally the succession of things revealed to us by imbedded fossils. Diastases are physical things in the same sense that molecules are. The distribution of the seas in the different geological ages is a system of causes which explains the distribution of land. Examples could be multiplied indefinitely.

The conclusions stated in connection with the physical sciences hold true for the natural sciences.

## CHAPTER XI

### THE VALUE OF OUR SCIENCES

According to the views already developed, any complete study of the value of our sciences should include two parts—the study of the edifice built up of the empirical rules and that of the physical theories which interpret them.

Empirical rules, one will say, are facts; they are, and that is all that can be said as to their value. “All that the scientist creates in a fact,” says Henri Poincaré, “is the language in which he enunciates it.” This is our point of view, except that to enunciate a fact is almost to create it entirely. It is to create it entirely, at least in the case of a scientific fact, which is the object of science. An unexpressed fact is but a vague representation, an object of revery which floats in the current of our thought and the boundary of which has not yet been traced. The representation of a fact may inspire the musician but it counts as nothing for the scientist. It is the proposition enunciating it that is everything for him.

We have already shown the reasons for this, the principal one being that only a proposition can be handled by that procedure of transformation which we call reasoning. An unexpressed fact gives the reasoning machine nothing to take hold of.

But there is more to be said. The expression of a fact gives it its existence by making it stand out from the current of sensations, by fixing what belongs to it and by assigning definite limits to it. It creates the fact by compelling us to consider it through the prism of our previously accepted conventions, explicit or not, by imposing upon it the actual form of our minds, by making it thinkable. The expression of a fact is itself an interpretation and we do not know exactly what separates that interpretation from the brute sensation. We have already shown the extent to which our empirical science would have been different if our rational ego had been different. We shall not discuss that further.

A single example will show that, by expressing a fact, one may sometimes create it entirely. When the unit, meter, was defined as one forty-millionth of the terrestrial meridian, the contraction of the earth had no meaning. By taking as the unit of length the standard platinum meter we gave birth to a new fact, the contraction of our globe, which can be measured. The day that unit becomes the wave length of a certain radiation, the contraction of the standard meter will have a meaning. Until that day, the platinum meter is invariable. Only wave lengths change. It is clearly seen from this example how the expression of a fact creates that fact.

Similarly, and in a more general way, we cannot conceive of a scientific fact without an observer to express it. Equations do not exist in nature any



more than colors or sounds. There is something, unknown to ourselves, which our ears translate into sound, our eyes into color, our mind into equations. Had our rational ego been different, the scientific image of the world would have been different, just as visual images for the color blind are.

Here again relativity confirms this view of the matter. A mathematical expression of a phenomenon is meaningless except as we determine which observer studies it. "Length and duration are not qualities inherent in the external world, but relations between the objects of this world and some definitely determined observer."<sup>1</sup>

Thus, empirical laws do not have any absolute meaning. They exist only for the mind which observes them, and have meaning for it only. Are they then arbitrary? Certainly not. They have been remarkably fruitful because of the power which they have put into our hands. They have yielded results the reality of which is indubitable. The telegraph, the railroads, and the aeroplane prove abundantly that there is something in those rules which we did not create. They prove to us that they are well adapted both to us and to the unknown world in which we are evolving. They are rules of action and as such amply serve their purpose.

This permits us to imagine how these rules were constituted. Their expression is the product of in-

<sup>1</sup> Eddington—Space, Time, and Gravitation.



numerable observations (often unconscious), of an infinite number of trials constantly repeated. When they were bad or when the mind which worked with them was poorly adapted to the purpose, the rules were sterile, and, in this slow, obscure, and confused progression, our mind was formed. All the rules which worked had a common character and it was from the ascertainment of this character that the laws of our reason were much later drawn.

These laws once established, the form of our knowledge was determined. Experience became susceptible of reduction to only a small number of expressions which constitute the laws of experimental physics. For beings such as ourselves, educated as we have been, and heirs of an ancestral experience such as ours, no other system of laws in the world in which we live was possible. How then must we evaluate the physical theories which interpret those laws? From their form, it would seem that these theories are designed to indicate the underlying nature of things, that which causes the empirical laws. They claim to be the exact description of the world in which we live and which they seem to take apart into its elements as the watchmaker does a watch. Atoms thus considered would be small bits of matter, possessing all the qualities attributed to them by the atomic theory. If our fingers were more delicate and slender, we should be able to take them up between the thumb and the index finger as we take up small marbles. To this conception the preceding

discussion opposes another one, altogether different; which, as we believe we have shown, establishes itself when we lay bare the mind of the man of science in order to study the ensemble of scientific theories.

The causes out of which we have made the "nature of things" are only a system of propositions designed to be the premises of reasoning whose conclusions represent the phenomena of experience. This "nature of things" is, in short, merely a device permitting us to substitute for the fixed succession of phenomena which we observe the causation which we desire.

Is then the construction of physical theories simply a play of the mind? We do not think so. The created causes permit us to deduce observed appearances, and, for this reason, they are not chosen at random. Furthermore, there is something in them that is not ourselves since, when these causes have a visually observable character, it is such that our senses reveal it to us. We have demonstrated merely the part that we play in ascertaining those coincidences and the multitude of systems of causes capable of explaining the same appearances.

When I observe a street scene upon the moving-picture screen, I interpret the aggregate of my sensations in a three-dimensional space. The street, the houses, the men walking away or approaching, are a system of causes which I have created with the aid of my previous knowledge and which explain the sensations that I feel. Thus the physical theory so

constructed is good. It continues to interpret without contradiction all the appearances observed. I even "see" the causes that I have created. Can I affirm that they represent the nature of things? Surely not. Let us try to determine precisely what, in this case, we mean by the nature of things, the true cause of our visual sensations.

Shall we say it is the screen, the point of departure of the luminous rays, which, provoking the appearances observed, contains the entire phenomenon? If so, we have translated into "our" three dimensional language a reality which has only two dimensions. We then have indeed constructed a physical theory in the sense that has been indicated above. But is the illuminated screen really the cause of the phenomenon? Is it not rather the luminous pencil of rays which strike it? Or, still better, is it not the combination of the film and the source of illumination?

We do not know and cannot know anything about it. We see clearly, then, from this example that the nature of things, the underlying reality, the logical cause of our sensations, are expressions which for us have no meaning, and can have none. We do not and cannot know more than a single category of causes for the appearances we observe, *viz.*, those that we create and which are the point of departure of our physical theories. We have seen the sense in which they can be said to be true. We saw the essen-

tially subjective criterion that presided over their choice.

This being true, an infinite number of systems of premises are possible. All interpret the aggregate of our sensations equally well. All could be equally true. How, among all these systems, do we choose the one to which we assign an exceptional place by decorating it with the pompous title of reality?

It is chosen because it is the system of causes which is for the moment the best both for us and for the world in which we live. It is the best because it is the product of life, the final outcome of our whole story. It is the result of the slow evolution that made us what we are. It is the best because it is. Other systems could be realities from only a rational point of view. This one is reality for our entire conscious existence. It was formed alongside that existence. It is normal for this system to have existence only by virtue of it. Were it not what it is, we should not be what we are. We thus understand why it is the best one, it having been made by us and for us.

Here again, the problem of the choice of causes seems insoluble merely because it is poorly put. Because several solutions are theoretically possible, our mind is imagined as hesitating at a given moment between several roads open to it, weighing the *pros* and *cons* and deciding only after mature reflection. This foolish conception is again a product of the two "I's"—the "I" which lives and the other which watches it living. We have shown that the

problem does not present itself in this way. The current of life is one—it embraces our entire being and nothing exists that could contemplate it. There is not, nor can there be, any conscious choice between equivalent solutions. There is, rather, a slow progression along a single path—that of life, and the construction of a single nature of things—the existing one. Other systems of causes that we might have created have no reality because they are only logical. In order for them to be true, it would be necessary for us to have been different—ourselves, and our past. For us, such as we are, only one system of causes is true, and that the existing one.

But, one may object, if our reality is merely a convenient interpretation among all the possible ones, how can it foreshadow new appearances? If it is not what is, how can it tell what is to be?

In order to answer this objection we must distinguish between two cases. If a present system of appearances has been observed before, it is normal for us to be able to foretell rationally what will succeed it. Reality, our reality, has been created for the sole purpose of making this prophecy possible. It is prudent, therefore, in such a case to act as if there were causes in nature and as if their true essence were known to us.

If, on the other hand, there is presented an ensemble of circumstances which is observed for the first time, there is nothing enabling us to foretell with certainty the system of appearances that will follow

it. In that case we can only note those appearances and attempt to draw from the premises previously adopted a proposition which will interpret the whole phenomenon. If the operation is possible, the reality remains good and it is retained. If not, it is modified, adapted, or replaced by new propositions which will permit us to predict the future with certainty.

It is thus seen that the further our empirical science is developed, the more prudent it is to base our actions upon rules rationally deduced from the reality of the moment. When the empirical branch of science has revealed only a small number of appearances, the reality adopted offers only a mediocre utility. When, on the other hand, this number is large, reality permits us to a great extent to foretell the succession of phenomena. Thus we have explained the formidable power that modern science has placed in our hands without there being any need to accord it any transcendental character.

Reality thus considered loses all metaphysical value. We have no means of affirming that the causes revealed to us by our physical theories are the true nature of things. Similarly if, as we have demonstrated, the Law of Causality is only a garb that we have placed upon the world, we have no power to affirm that there are causes in nature.

The only metaphysical lesson we can derive from the existence of our sciences is that their construction has been possible. The laws of our rational ego, those of Identity and Causality, the implicit axioms



presupposed by our language, our methods of determining distance and time, have all rendered possible the construction of physical theories which have interpreted the succession of phenomena of experience. They have been found to be good tools, both for us and for the unknown something that determines our sensations.

Establishing this fact, perhaps, throws a feeble light upon the manner in which they were formed—results also of vague intuitions, of unconscious observations, becoming incorporated with the flow of an intellectual life scarcely formed, of experiences in infinite number. We are here entering into the realm of hypothesis; we shall not remain there. Suffice it to establish that these rational laws are today the very conditions of our thought, so incorporated with our reason that they are reason itself. To attempt to think without them or against them may be possible in the mathematical sense of the word, but it is not true.



## PART THREE

### THE SO-CALLED SOCIAL SCIENCES

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#### CHAPTER XII

##### GENERAL CONSIDERATIONS

We shall find in the social sciences all the characteristics of the sciences properly called "scientific." Each of these sciences includes, as we shall see, an empirical branch which gathers appearances and expresses their common characteristics in the form of laws; and a rational branch which creates their causes. These causes, as in the physical sciences, are a system of initial propositions, axioms and definitions capable of serving as premises to reasoning whose conclusions coincide with the laws empirically discovered.

We shall be obliged in the following chapters to keep constantly in mind all of the preceding ones. We in no wise attempt to modify the social sciences in order to bend them to the scientific form. We affirm, on the contrary, that, just as they are now, they present all the characteristics of the so-called physical sciences. Nor should this astonish us since, as Descartes said, intelligence is one, as is the sun for the worlds it illuminates.

There is no one science but sciences all using the same instrument under one of its two forms—formal

logic or analysis. So has it been for all the sciences thus far studied. So will it be for those which remain to be considered.

The current of thought taken by itself and in its totality, independently of what it represents to us, belongs to the domain of psychology.

By choosing within this current certain categories of representations, we have fixed the limits of the sciences. The empirical branch of mechanics, for example, deals with those of our sensations which reveal movement to us.

Now, in the current of thought, there exist certain elements which reveal to us the "ought-to-be." It is their study that constitutes the empirical branch of ethics.

Other representations apprise us of facts relative to the functioning of our society. These are the subject-matter of empirical political economy.

At first sight, these two sciences seem to be of a very different nature. Although political economy, studying what is, can be called a science, morals, studying the "ought-to-be" seems to be only an art.

This conception can be based only upon a misconception of the facts studied by empirical morals, for there surely exists in the current of thought a category of representations telling us what ought to be. These representations are facts just as much as those which reveal to us the forms of our society. Both the latter and the former are, under given conditions and at a given moment, universally observed.

Alongside of these sciences, namely political economy and ethics, there is an art, just as alongside of thermodynamics there is an art of constructing motors. The art or the application of political economy determines the best conditions of production, for example. The art connected with ethics tells us to what extent the rules of the ought-to-be should direct our lives. They are both uncertain and susceptible of debate. Not so with the empirical branches of ethics and of political economy. This we shall see in the following pages.

From those representations, which are moral or economic facts, we derive laws. These laws, as in the physical sciences, express the common character of a series of representations. Their existence has often been considered doubtful. We shall see in our special study of these sciences that it should not have been. If these laws appear uncertain to us, it is because we regard them too often from a purely subjective point of view. Moral or economic facts are intimately connected with all our interests, all our passions—and such conditions are quite unfavorable to scientific study.

But, above all, the principal proof of their uncertainty is taken to be the frequency of their variation. History seems to reveal to us an unstable morality continually modified, economic or social laws always changing.

We shall show that such is not the case at all. Moral or economic laws are immutable but they do

not all operate at the same time. A law is the expression of a sequence of events. Thus, in the present state of life, such and such circumstances are present and such and such laws permit us to foretell phenomena resulting therefrom. It is the knowledge of the initial conditions that makes us choose in the arsenal of empirical laws those which it is fitting to apply. And if, in the course of the evolution of our world, certain circumstances should cease to be present, the laws permitting us to deduce therefrom the resulting phenomena would evidently cease to apply. They would no longer have any current reality but they would be none the less true.

Thus slavery or monarchy by divine right gave rise to certain moral laws relative to the duties of individuals. Production within the family or the absence of rapid means of communication revealed to us certain economic laws. Those conditions of life have disappeared or are in the process of disappearing. The laws which appertained thereto no longer apply. Are they, for this reason, any the less true? Evidently not. If the new means of transportation should happen to disappear, undoubtedly the ancient laws would once again come to rule those conditions of life; and the conditions of dearth, which those laws made it possible for us to foresee, would again appear on the earth.

Thus laws remain true but the conditions under which they apply may or may not be present. This shows why the laws of the social sciences seem to

belong to only one period, although actually they are as immutable as any body of empirical laws.

The physical world, moreover, presents the same phenomenon, making the meaning clearer. According to whether the sky is overcharged with clouds or clear, the laws of atmospheric electricity will or will not apply. Depending upon the temperature or the condition of the atmosphere, the laws of the condensation of vapors or those of evaporation of liquids will govern the succession of meteorological phenomena. If we must know the laws of thermodynamics in order to foresee the operations of a locomotive, we shall utilize those of electricity when the line has been electrified. The laws of the steam engine will remain true but they will no longer apply in the restricted universe that the railroad line under consideration becomes.

Thus, the laws of the social sciences take on their true worth. Now the question presents itself as to what force replaces the steam locomotive by an electric locomotive, the dark heavens of stormy weather by the azure of beautiful days, the monarchy by the republic, the domestic workroom by the giant factory.

That force is life, the entire life of the universe, the life which goes on, irrespective of ourselves and because of ourselves, the vast synthesis of all being, the infinite progress whose end we do not know. It is life that in its course determines each stopping point, that implies its appropriate laws. We hear it

rumbling all around us. We feel confusedly the clashing forces out of which the future will be born. Revolutions as well as evolutions are the results of it. Their causes are infinite in number. It is this immense movement which determines at each epoch the reality of the moment.

Can the course of this movement be foretold? Can the evolution of the world be determined? We do not think so.

Our empirical laws are, for one thing, differential. They all express an instantaneous phenomenon, the passage from the antecedent to its consequence. The cases where they can be integrated are infinitely rare.

But more than that, the antecedent is never more than an approximation. It was artificially isolated from the current of our sensations and so permits us, the law being supposed unknown, to foresee only a fraction of the consequence. Now the evolution of life is engendered by the immense complexity of things. The factors that determine it are innumerable. No science can, therefore, claim to foresee the entire evolution of our universe, to guess at what moment a certain physical or social law will cease to apply.

An example will help us better to understand this. The adoption of the relativity point of view obliges us to take account of the fact that Euclidean geometry of three dimensions no longer expresses the properties of sensible space. The processes of our sensa-



tions, however, have not been modified, merely the laws derived therefrom. This modification can be attributed only to our minds, to our methods of observation and of calculation, and to the presence of the man of genius who perceived the fertility of the new relations. It was, therefore, entirely unpredictable. Empirical science must content itself with establishing its presence by recording the new laws it establishes.

It is the same in the world of human society. It is present social conditions which determine the empirical laws that permit us to foretell phenomena at a given moment. If it is the law of supply and demand that expresses the common character of exchange in France, it evidently does not follow that it will be the same in Russia. Now, the evolution of the social state is in the scientific sense unpredictable. Empirical economic science must be content with recording the laws which apply in the social state attained by the total evolution of the universe studied. It can never affirm that a certain law will not become true.

Could this evolution have been different? We do not and cannot know anything about that. The very question is devoid of meaning for us. All we can say is that the state of our universe is completely determined at each instant and this suffices also to determine at each instant those laws which constitute the empirical science of the moment.



We shall study particularly among the social sciences, ethics and political economy. The latter science, very much developed in its present state, will display with great clearness all the characteristics of the physical sciences.

When we have proved the analogy which we have been foreshadowing, we shall derive therefrom all the instruction it will permit. We shall see that it is rich in practical consequences, quite as much in the exposition of the social sciences and the form we give them as it is in determining the value which we should ascribe to them.

## CHAPTER XIII

### PSYCHOLOGY

Psychology in its present state is characterized by the fact that its empirical branch is very much developed and its rational branch very little.

The empirical branch of psychology is that which attempts to discover the laws governing the current of thought, taken in all its aspects and in its infinite variety. The rational branch aims to interpret the laws thus discovered, *i. e.*, to create their causes.

The first physical theory of psychology was based upon the affirmation of the existence of the soul. The physical entity thus created explained the appearances observed but with very little precision. Having shown itself, in the course of events, to be insufficient, it was replaced by theories which constitute modern psychology, each of them explaining only a limited number of appearances. We shall take one as an example—the theory of the subconscious.

By its very nature, the subconscious is unobservable. It is not consciousness, therefore, which reveals its existence to us. We are led to admit the existence of subconscious psychological phenomena in order to explain certain psychological phenomena, normal or pathological, revealed to us by observation of the current of thought. The subconscious is the simplest

and most perfect cause of those phenomena. "We assume the existence of unconscious psychological phenomena," says M. Roustan,<sup>1</sup> "when certain conscious psychological effects, carefully noted by physicians and psychologists, oblige us to accept that hypothesis as the one which best adapts itself to the facts, and is, at the same time, the most economical and the most fertile."

We purposely quoted these lines in order to show that it is impossible to conceive of the subconscious other than as a cause created by ourselves in order logically to explain an aggregate of phenomena empirically observed. Here, as in the physical sciences, the theory is a system of causes. Again, as in the physical sciences, the created cause, *i. e.*, the subconscious, coincides in those parts where it can be observed with what is revealed to us by consciousness. The penumbra surrounding consciousness, the fringes which shade off its confines, would thus be the limits of the subconscious. Thus, the created cause, in those parts in which it is observable, is, in fact, observed. The theory is good.

Let this example suffice to show us the nature of the rational branch of psychology. In all cases, it is possible to reduce its theories to a creation of causes, and to show that its form is rigorously comparable to those of the sciences which we have already, studied, *i. e.*, the sciences indisputably scientific.

<sup>1</sup> Psychologic, p. 74 (Delogrove, publisher).

## CHAPTER XIV

### SYSTEMS OF ETHICS

The empirical branch of ethics is that which enunciates the innumerable rules of morals, the rules of the "ought-to-be," which are, in fact, the practical morals of the moment in which we live and for our particular group. These rules constitute an aggregate of commandments purporting to limit the course of our actions: thou shalt not kill, thou shalt not lie, thou shalt respect the life of thy fellow, thou shalt love thy parents, thou shalt be upright, generous, charitable, industrious, temperate. . . .

It is an indisputable fact that many of our notions possess the character of the "ought-to-be." They constitute the moral facts whose existence can no more be doubted than can that of physical facts. Now these moral facts may be grouped into categories. Moral laws are limited to expressing the common character of the facts included under these several categories. They, just as physical laws, are drawn directly from the current of our thought. They express moral reality.

These moral rules are, during a given epoch and in a particular environment, almost universally admitted just as are the empirical laws of the other sciences.

The fact that the conduct of individuals respects these commandments to a greater or less extent does not in any way diminish their objectivity. As moral facts, they exist only as rules of the "ought-to-be." They do not express what is.<sup>1</sup> The individual who violates them does not deny their existence. He simply does not submit to them. We cannot claim that theft does not exist<sup>2</sup> but we can affirm that the maxim, "Thou shalt not steal," is common to all representations of the stream of our thought. Thus the system of moral rules does not direct our action, it *tends* to direct it. It tells us what it ought to be in order to be "human." In this sense and in this sense only is it universal and necessary as are the systems of empirical rules of the physical sciences.

We could repeat regarding the rules of ethics all we have already said of scientific rules. We have seen, and need not state again, how and why the rules of mechanics are universal and necessary but might have been different. The considerations developed at that point apply precisely to the rules of practical morals. They are universal and necessary but are so only for and through ourselves. They are the product of the education received, of the principles adopted, of the present social state and of all those which preceded it. They are a function of all our attainments, of all our faculties, of our entire

<sup>1</sup> But compare viewing the behavior of matter as statistical averages of the behavior of its molecules, for instance.—Ed.

<sup>2</sup> But we can affirm that most men do not steal.—Ed.

history. They are the product of life. To reconstruct their slow discovery, their slow growth, would be to reproduce step by step the entire history of humanity, to evoke all its phases and all its factors, and to depict the constant progress which brought it to its present form. This is an impossible task because the number of elements is infinite, each one reacting upon all the others. Ethics can no more be conceived as detached from political and social history than from economic or food conditions, from climate or from the fertility of the soil.

The first man probably had no more idea of duty than of causality. He lived—and that is the key to our history. He lived and life made him what we are. By what process of evolution, by means of what intuition followed by what unconscious experiences, repeated indefinitely, he arrived at the notion of duty and of right is something that we can only imagine. The constant action of the world upon his growing mind, his unceasing struggle with the elements and against his own kind, the ideas derived from all that made him aware of his own feelings, the necessity of having a plot of land and of retaining it, fear, pain, love, hate, his entire physical and moral nature and the infinite complexity of the world, all these must have engendered within him, little by little, the idea of the “ought-to-be.” The unfortunate man constantly discovers the rules of morality. He perceives their character and their necessity. When he has been robbed by means of a lie, he discovers that one

should not lie. The abandoned father discovers filial duty. The man who fears highwaymen discovers the right to life.

Thus the rules of ethics cannot be conceived as isolated from, and independent of, the entire life of our universe. They are its result, its inevitable outcome. They have no absolute meaning.

The existence of these laws might perhaps be connected up with the great theories of evolution and of adaptation to environment. So conceived, they would be, at each epoch, the most efficacious means of assuring the greatest happiness to the greatest number. They would be for humanity the means of realizing its ends. But we are here entering into the realm of speculation. We shall not tarry there. Moral laws exist, and, from a scientific standpoint, that is all we are interested in. They have an undoubted existence. We are not at liberty to modify them. They constitute a moral reality, an aggregate of laws tending to direct our action just as surely as does the physical reality of which we have already spoken. Does this mean that they could not have been different? We do not think so. If the laws of our mind or if our history had been different, our moral laws would not be what they are. The evolution of these laws in the course of the centuries proves abundantly that the morality of each epoch is an integral part of the reality of the moment. The morality of a monarchy is not that of a republic. Revolutions, modifying the conditions of environment, create new



rights and duties. The abolition of slavery and the disappearance of the feudal régime have revised the rules of empirical morals. Each social transformation brings with it its train of new laws. As for the cause determining these changes, we have described it in the preceding chapter. It is the total life of our universe. It constructs and it remodels without end. All around us we feel life evolving, new conditions coming about, laws asserting themselves. Codes voted by parliaments are, when good, merely the expression of a widespread sentiment in regard to the "ought-to-be," to which a new situation has given rise.

Thus moral laws exist. They are natural laws. That they, just like physical laws, are the best for us and for the world we live in is revealed to us by the wail of victims which arises when these laws are violated.

This moral reality once known, we must, following the general plan of constructing a science, "create its causes," *i. e.*, explain it for our human minds. In order to do this, we, as usual, enunciate a system of initial propositions, axioms and definitions which, when fed into the reasoning machine, will produce theorems coinciding with the rules of practical morals. In other words, we create a moral geometry.

We shall, in the following pages, make a special study of some of these geometries. We shall rediscover in them all the characteristics of the rational branch of the physical sciences.

We must first of all note that, just as in the physical sciences, an empirical moral law exists only after it has been expressed. Anterior to the proposition enunciating it, there exist only vague appearances, only desires having the same vague character. There is no moral law in the scientific sense of the word and this for the reasons already developed. An aspiration can no more be adjusted to the working of the reasoning machine than an image can. It mills only words, not appearances.

Furthermore, the theorems of a physical theory are propositions. In order to judge of their coincidence with empirical laws, those laws must first be translated into propositions. The entire art of the student of ethics consists of expressing by means of words moral rules of which we all have a vague intuition. It is because this work is done only rarely and not very rigorously that theories of ethics are so uncertain.

Once the rules of empirical morals have been rationally found, the reasoning machine, just as in the physical sciences, exceeds by far the limit that had been assigned to it. By the deductive method, it draws from the initial propositions theorems much more numerous than the empirical laws of morals which it had been originally intended to explain. These theorems constitute a part of ethics. So long as the rationally deduced theorems are not in contradiction with the rules of the "ought-to-be" extracted from the current of our thought, the physical

theory is good, the causes are retained, and we say that they represent the nature of things. When the coincidence is no longer present, the initial propositions must be changed. They must be adapted when their adaptation is possible. If we do not succeed in doing this, we must overturn the edifice and give it new bases.

The succession of moral theories in the past is the history of these renewals constantly effected in order rationally to explain new customs. Thus is explained the fact that nations have always had that system of morals which justified their current rules of life. Moral theories no more make customs than do our ideas about the constitution of matter make the properties of bodies. Both the former and the latter are created for the sole purpose of explaining a system of appearances which is given to us. A few examples will illustrate this statement.

Greek society was based upon slavery. It was an established institution. In order to make of it a rational necessity, an axiom was enunciated wherefrom there could be logically deduced the manner of life imposed upon that class of men called slaves.

In fact, Plato says in his "Republic" that "in the mind of slaves there is nothing sound or whole and a prudent man would not rely on that class of men." Again, Aristotle affirms that "nature created the bodies of free men different from the bodies of slaves since she made the former for commanding, the latter for obeying." Thus, the necessity of

slavery may be logically established once we admit either of these two axioms.

Similarly, absolute monarchy was the established political state in France of the Seventeenth Century. Now, at that epoch, we have a physical theory purporting to explain all phenomena of experience, to wit, the religious theory based upon the axiom of the existence of God. In order to make of absolutism a rational necessity whose existence the physical theory may foreshadow, a created cause is adopted. It is affirmed that "royal power is of divine right," and this auxiliary definition, when joined to the characteristics of divinity, contains potentially all those of absolute power. They need only be extracted by the reasoning machine. Revolution supervenes. The power passes to the people. That is now the established fact. Immediately there is created an axiom which will serve as the foundation of a theory for the new state affairs. "The principle of all sovereignty resides essentially in the nation; no body, no individual may exercise any authority which does not expressly emanate therefrom." (Art. III of the Declaration of the Rights of Man.) This axiom being considered as expressing the nature of things, democracy becomes a rational necessity. As for revolution, that is life rolling on, with its crises and its storms, making our reality.

Thus is the nature of our explanations made clear. Life moves along. It brings into play innumerable forces confronting us on all sides. In the immense

crucible which our universe is, life is at each instant forging the reality of the moment. We, far behind, follow painfully and slowly. We discover laws expressing the characteristics of this reality. When they have been enunciated, we make out of them for ourselves, and for ourselves only, an explanation which rationally justifies their existence. We create the causes of these laws in the logical sense of the word.

#### ETHICAL THEORIES

Thus conceived, ethical theories are, as are all the physical theories, rational systems constructed from causes which we create in order to rediscover by the deductive method the empirical laws, which life has compelled us to recognize. They are, therefore, geometric systems of ethics and they have, accordingly, just as does the geometry of space, the strictly logical and rigorously determined character of purely deductive structures. Moreover, the only means at our disposal for modifying the theorems of such geometric systems, is to modify the initial propositions which serve as their foundations. In order to bring these theorems into coincidence with the rules of practical morals, we must adapt the causes or replace them when their adaptation is impossible. Thus, by modifying some one of the initial propositions of a system, we cause all the theorems of a geometric system of ethics deduced from those propositions to vary.

We are thus brought to distinguish between two groups of ethical theories; Euclidean morals, whose theorems coincide with those empirical rules which life has compelled us to recognize, and non-Euclidean morals, rational structures, one of whose initial propositions is in contradiction with some one of the initial propositions of a Euclidean system of morals.

Furthermore, we have seen that, given a certain number of propositions, it is possible to create several systems of premises capable of being their rational causes. These systems of premises are, from the logical point of view, equally good. One of them will be "more true" than the others only if it explains a larger number of theorems or if it connects up more easily with the ensemble of physical theories already adopted. Thus the truth of moral theories is deprived of all absolute value. A theory is not true absolutely or inherently. It is merely more or less true according to the epoch, the aggregate of the system constituting the theoretical science of the moment; according to the individuals, their conditions of life, and the sum-total of their aspirations.

One thus understands why there is an abundance of ethical theories. The empirical rules which it is proposed to deduce not having been systematically enunciated, each individual is led, according to his tendencies and his social environment, to attach more importance to some one or the other of these rules, at the expense of the rest. The causes which for him constitute the nature of things will often have no



other end than to explain and justify those empirical rules which he regards as paramount. Wherefore the prime importance of a systematic construction of ethical theories capable of explaining with the least number of initial propositions the greatest possible number of rules of empirical morals.

We are now going to examine in succession some of the existing ethical theories, Euclidean and non-Euclidean. We do not propose to make a detailed study of them, nor a critique, but merely to give some examples of the preceding considerations.

These theories were all constructed in the same manner as were the physical theories already studied, but less rigorously and under a less systematic form.

#### A.—EUCLIDEAN SYSTEMS OF ETHICS

##### 1. *Theological Systems*

The theological systems of ethics go back to a general system of explaining the universe in which system the divine will is the first and universal cause of all the appearances revealed to us by our senses. The created cause is God; its definition, the untrammelled arbiter. We have already shown the characteristics of this system of causes, created in the image of the only cause known to us, our wills.

From the moral point of view, the two axioms which serve as the premises of the theory are the following:

“Man should do what is good.”

“Good is what God has willed.”



Whence it results that man ought to do what God has willed. A logical system is thus constructed. The sum-total of the commandments of God, handed down to men by means of a supernatural revelation, should regulate all the acts of their lives.

Now, these commandments of God coincide in general with the rules which life has compelled us to recognize. In this way the divine will suffices to explain the totality of empirical laws. Being the first cause, it is the cause of phenomena of experience. From the purely logical point of view, the physical theory which can be built upon it is sound.

## 2. *Sentimentalism.*

This is an ethical theory resembling the theological theories with respect to the nature of the cause it creates, *viz.*, the moral sense, which distinguishes good from bad just as the eye distinguishes day from night. God, instead of handing over to us a long list of our duties, gave us a sense capable of revealing to us in each case what was right and what was wrong. "Conscience, conscience, divine instinct, immortal and celestial voice, sure guide for a being, ignorant and limited yet intelligent and free, infallible judge of right and wrong, which makes man resemble God." (Rousseau.)

The existence of the moral sense is an axiom which makes it possible to explain in each case at a given time the current feeling as to the "ought-to-be," from which we have derived our moral laws. But

it does not explain the variation of these laws in time because the moral sense evidently must remain unchanged indefinitely. It is, therefore, an insufficient theory.

With this theory of the moral sense should be grouped those systems of ethics based upon the existence in our hearts of a sentiment which pushes us toward the good. Whether this sentiment be sympathy, as it was for Adam Smith; pity, as it was for Schopenhauer; or altruism, as it was for Auguste Comte, it is, in any case, the affirmation of the existence of a principle from which can be rationally deduced the rules of our practical morality, the only moral reality known to us. It is, therefore, still the creation of "moral entities," capable of being the causes, logically speaking, of the appearances constituting that reality.

If causes as different as sympathy and pity made it possible to justify the same laws, it is because there was joined to the axiom affirming the existence of such a sentiment a host of other implicit axioms upon which these moral theories rested and further because no attempt at rigor characterized the deductive reasoning used in building them up.

### *3. The Kantian System of Ethics*

Kant's system displays with great clearness the characteristics of a geometric system. It is a deductive edifice entirely, constructed from axioms and abstract definitions as the starting point, and it is

very noteworthy in its support of our thesis that all the consequences which may be drawn from his principles coincide with the appearances revealed to us by the observation of the stream of our thought, such as the notion of duty or the rules of practical morality.

In order to explain the notion of duty, Kant assigns to our reason "pure elements" which make possible the determination of moral law *a priori*. By coming into conflict with the tendencies of the individual, moral law takes the form of a categorical imperative which gives rise to the sentiment of duty. Thus the cause, created in order to explain the existence of this sentiment, is at bottom a form of our reason. Then he has to deduce the rules of practical morality. To do this Kant creates a "moral being" and good will, and he enunciates the following axioms:

"Good will is that which acts out of respect for moral law."

"Good will alone may be held to be morally good."

And, since it is not sufficient for the moral law merely to exist, it having to be known to us, he says:

"The moral law is defined by its conformity to the very idea of law, that is to say, the maxim or subjective rule of the will must be such that the subject can will that it be set up as a universal law."

From those three axioms Kant derives the three formulæ of the categorical imperative, from which

the reasoning machine is able to derive all the rules of practical morality. It is thus seen that the axioms quoted are only a means of rediscovering those laws.

#### *4. The Hedonistic and Utilitarian Systems of Ethics*

The construction of these systems is again the search for a principle which will contain potentially all the rules of practical morality, which it will be the function of the reasoning machine to extract. For all these systems "the value of an action depends solely on whether it excites within us a determinate affective state, whether it shows a special emotional character."<sup>1</sup> The axiom which embodies this character is the base of each of these theories. The "good" is thus that which affects us in a certain manner and the existence or the absence of this emotional character will determine in each case what ought to be.

It is now possible to see what the form of all the syllogisms which are to rediscover the rules of practical morality is:

"A certain act creates within us a certain affective state. Now any act which creates that state within us is good. Therefore, this particular act is good."

Such is the geometric system of morals, simple and rigorous, which will justify for our minds the rules of the "ought-to-be."

The created cause that explains everything is the second proposition of the syllogism, "Every act

<sup>1</sup> Malapert—Course in Philosophy, V. II, p. 41.

which creates that state within us is good." It is in the enunciation of this initial proposition that all the varying degrees of freedom associated with this system are found. This choice once made, the entire system is determined.

The motives directing this choice are known to us. The axiom enunciated must make it possible rationally to deduce the rules of practical morality. The affective state chosen as the criterion of the good, will depend, therefore, in a general way, upon the rules which tended to make themselves known at the epoch of its choice. Thus is explained the existence of these systems of morals.

To enumerate them, we find first the ethical systems of pleasure. Pleasure is the only test of the good, pain the only test of the bad (Aristippus of Cyrene). Thus are legitimized the rules of life of the Cyreniacs. Their causes are created. The pursuit of pleasure is thus rationally explained and justified.

With Epicurus the level of the pleasures sought is raised. The criterion of the good is happiness or pleasure properly understood, which means understood after the fashion of the Epicureans. Thus is legitimized the Epicurean mode of life.

With Bentham the system is complicated. It becomes an arithmetic of pleasures. The quantity of pleasure arising from an act is a function of seven quantities, *viz.*, intensity, duration, proximity, cer-

tainty, purity, fecundity, and extent. Each of these characteristics has a certain numerical value, either positive or negative. An act is good or bad depending on whether the sum of these coefficients be positive or negative.

We are stressing this theory somewhat because it shows us the creation of causes in one of its most characteristic phases. Bentham recognized that an act is the more desirable the more interest it has for its author, and the greater the number of individuals participating in the pleasure resulting from it. Pleasure is for him no longer sufficient to determine the good. Furthermore, because of his tendencies, his environment, his education, or something else, he adopted a morality of pleasure which no longer accounted for the phenomena he observed. Rather than renounce his theory, he attempted to adapt it to newly observed appearances. He created auxiliary initial propositions, added some new characteristics to the pleasure that an action must bring in order to be good. He made his theory so complicated and artificial that, in considering it, one has a very clear impression of watching the effort to adapt causes, a process which precedes their replacement. The theory of Bentham is instructive because of its very naïveté. The process is visible at each step and one feels sure that this curious arithmetic was only invented to justify the rules of practical morality.



With Stuart Mill the criterion of a good act is its conduciveness to the greatest happiness of humanity. That is the axiom which is basic to his whole theory.

“This act is productive of more happiness than that.  
Now any act which produces happiness is good.  
Therefore the act is good.”

In order to be able in each particular case to enunciate the first of the propositions of this syllogism, a means of comparing the happiness resulting from two different acts is necessary. For this purpose Stuart Mill enunciates the following rule:

“If one wishes to know which is the better of two pleasures or which of two modes of life gives the greater happiness, one should have recourse to the judgment of those who have tasted the two pleasures or tried several modes of life.”

This judgment, according to him, gives preference to those modes of living which employ “the most elevated faculties.”

With Spencer the axiom which becomes the basis of his entire moral theory is the following: “The purpose of society is the preservation and the happiness of the individual.” This proposition should make it possible to rediscover the rules of practical morality. Now it is not sufficient to do this, for many of these rules are more concerned with the welfare of the group than of the individual. In order for these rules to become a logical necessity, Spencer adds to the fundamental principle an auxiliary proposition which affirms that the perfect life of the individual is conditioned by the highest social life and



that, when these two ends do not harmonize, the latter becomes the more important.

These two axioms taken together makes it possible to deduce all the rules of practical morality, whose causes seem thus to have been found.

We are far from having exhausted the list of moral theories. They are innumerable. Plato, Aristotle, Malbranche, Leibniz and any number of others have all had their systems.

This abundance should not cause us to wonder. We saw in our study of syllogistic logic that there may be several systems of premises leading to the same conclusions. Physics shows us that the theory of emission and the undulatory theory of light have a large number of propositions in common.

In morals it is even more natural for the theories to be numerous since other reasons are joined to those we have just mentioned. The geometric systems of ethics were not constructed as geometries were. Their authors did not regard them as physical theories and were not careful to make their deductive character evident. Hence the want of rigor in their exposition. The initial propositions, not very rigorous, are not systematically enunciated. Their number is not reduced to a minimum. Many are not even explicit. Furthermore, the laws which it was proposed to deduce were not clearly enunciated. The entire edifice is fragile. Why, then, should we be surprised that here a great number of systems of initial

propositions lead to the same set of conclusions; that a large number of ethical theories may be approximately true, but no one of them entirely so?

#### B. THE NON-EUCLIDEAN SYSTEMS OF ETHICS

If ethical theories are really geometries, we should be able further to pursue the parallel. There should exist alongside, the Euclidean systems of ethics (rediscovering the rules of moral reality whose existence life has compelled us to recognize) non-Euclidean systems of ethics, rational edifices constructed by deduction and starting from axioms one of which, at least, must be in contradiction with one of the axioms of a Euclidean system of ethics.<sup>1</sup>

The right to live one's life is such an axiom. The practical rules which may be deduced from it contradict the rules of our empirical morality just as the theorems of the geometry of Lobatchewsky are in contradiction with the rules of surveying.

Thus considered, immorality is to morality what the non-Euclidean geometries are to that of Euclid, *i. e.*, theories rationally true, but humanly false. "Immoral theories" are logical edifices and nothing more. They have no point of contact with present

<sup>1</sup> It is brought to my attention that the word "Euclidean" has a very special significance and serves to characterize the geometry which is based upon the postulate of Euclid. It is evident that I am giving it a much more general meaning by calling Euclidean science any science which rediscovers the empirical laws of the universe at a given moment, as the geometry of Euclid rediscovers the empirical laws of space.

reality since their premises were not created to the sole end of rediscovering its laws. As such, they are incapable of directing our action. Are these systems of ethics then nothing but mental gymnastics? We do not think so. Like the non-Euclidean geometries, they may become true some day when life has set up new empirical rules.

The present morality of nations is a non-Euclidean system. Its axioms were not chosen with the intent that they serve as premises for lines of reasoning rediscovering the rules of moral reality. They were posited *a priori*. The morality derived from them is, therefore, a rational structure, but it is not true. It may become so if some day the entire life of our universe should make a reality of a society of nations.

It has been possible in this connection to verify the fact that theoretical ideas did not create a new moral reality as the sculptor fashions a statue. We have been able to observe that the material world did not let itself be molded to the rules formulated by President Wilson, deduced from *a priori* axioms. Thus, these axioms, even though universally recognized, did not become the causes of new appearances. They cannot become so unless those appearances are some day made a reality by the progress of life. But they will then be only their logical causes, causes for our human minds.

Thus, in short, the Euclidean moral systems have all the characteristics of physical theories. They

rest upon a system of causes created in order to be the premises of reasoning whose conclusions coincide with the rules of practical morality, rules the recognition of which life has forced upon us and which constitute moral reality.

With this fact definitely established, it follows that it would be very advantageous to expound ethical theories like true geometries, to systematize the different phases of the creation of causes.

To do this it would be necessary first to express with great clearness the empirical rules which constitute the moral reality of the moment.

We would then enunciate, in conformity with the rules of Pasch,<sup>1</sup> the axioms and definitions upon which the theory must stand.

These premises posited, we could derive from them by means of rigorous reasoning propositions which would have to coincide with the empirical rules previously enunciated. After having made sure of this coincidence, we could develop the theory by deductively deriving from the initial propositions all the theorems that they entailed. We should compare each of these theorems with the corresponding facts revealed to us by observation. As long as there was no incompatibility, the theory would remain true. In the contrary case, it would be necessary to modify it and, if this were found impossible, to replace that system of causes and adopt a different system.

<sup>1</sup> Chapter VII, the Geometries, p. 28.

Similarly, whenever a new empirical law was enunciated, it would be necessary to attempt to rediscover it by the deductive route, adding, as need arose, to the initial propositions a new proposition. If the operation were found to be impossible, the adopted theory would become false.

This mode of exposition would undoubtedly eliminate a certain number of systems of ethics. A number of them would, however, remain, and then only reasons of convenience would enable us to choose among those systems, they being logically equally valid. Those theories which were based upon the smallest number of axioms and which would best connect with the edifices built up by the related sciences would clearly be the truest.

Does this conception of moral theories necessarily entail scepticism? We do not think so. A system of causes being retained only so long as it accounts for the laws empirically discovered, it is the part of wisdom to base our actions upon the consequences which may be derived therefrom, and this all the more so since a system of ethics entails a greater number of laws. Thus from a practical point of view, everything would take place just as if there were a nature of things and as if the system of causes adopted corresponded to its essence. Our curiosity would be satisfied. Our human minds would have comprehended the world.

## CHAPTER XV

### POLITICAL ECONOMY

In its present condition political economy is an advanced science. Many of its chapters have the rigor of a rational theory. We shall, furthermore, be able to see therein with great clearness the characteristics which we have said were those of any science whatever it may be. As in all the sciences, the empirical branch of political economy collects the appearances which it obtains from observation of the external world. It enunciates laws expressing the common characteristics of a certain group of these appearances.

One needs only to open a book on political economy to ascertain that a large number of the empirical laws thus enunciated relate to the prices of goods and of labor and to the variations of those prices. Therefore, a knowledge of prices is an essential condition of all economic study.

It may be said that the existence of price is a fact. Commodities have a price just as lengths have a measurement. We shall not pause over the difference between the price of an object and the measurement of length, but we have seen that our method of measuring length and time and magnitudes in general is not imposed <sup>1</sup> upon us, but is chosen by us and that

<sup>1</sup> Chap. VII and VIII, Geometry and Mechanics.



it might have been different. We have seen that upon this choice depends the expression of the empirical laws of geometry and of mechanics, and that, consciously or otherwise, it was made only in order to reconcile the aggregate of our experiences with the principle of Causality. We shall see that the measurement of prices presents these same characteristics.

Simple observation of the external world teaches us that we can procure objects only through the process of exchange and that the price of an object is the number of units of price that one can get, or that one must give, in exchange for that object. The method of measuring prices being defined, we should note that it is as rigorous in its form as that of measuring temperature, for example. The number expressing the price of an object depends upon the choice of the unit of price, just as temperature depends upon the choice of the thermometric substance.

Now the money actually employed is gold, that is, a commodity difficult to procure. The unit of price is thus the value of a certain number of grams of gold. The use of this unit is in no wise forced upon us. We could just as well have taken as the unit of price the price of a hectoliter of wheat, but it is quite evident that in that case the expression of our empirical economic laws would be modified. It is thus seen that this problem presents itself here just as it does in the choice of the units of length or time in mechanics.



We shall inquire into the reasons why we adopted gold as the monetary standard just as we have, up to the present, adopted the substance of bodies called solids as our units of length. Suppose, for example, that we had chosen as our unit of price the value of a hectoliter of wheat, and that for two consecutive years the harvest of wheat had been very different, very abundant the first year, very much reduced the second. Let us consider under this system of measurement the price of some object whose economic conditions seem to us to have remained unchanged during the two years both from the point of view of the existing quantity as well as from that of production and consumption, gold for instance.

In order to obtain a kilogram of gold, we shall have to give less wheat the second year than the first. The price of gold will have diminished. We see immediately that this fact is in contradiction with the Law of Causality, identical antecedents, *i. e.*, all those which are tied up with gold (and we have a very definite idea of those antecedents) producing different consequences, *i. e.*, the two different prices of gold for those two years.

If, on the contrary, we adopt the value of a gram of gold as the unit of price, the price of a hectoliter of wheat will have varied with the conditions attending its production, and we are able to find in this variation an explanation consistent with the principle of Causality. The value of the gram of gold will be, for several consecutive years, a unit of price

permitting us to extract, from the phenomena of experience, laws compatible with the principle of Causality.

But if, for the same commodity, we compare its prices stated in gold coin for periods several centuries apart, we find a general increase in the price of all objects. This appearance is again in contradiction with the principle of Causality since different antecedents, *i. e.*, the conditions of production and consumption of each object, have produced the same consequence, *i. e.*, a general increase in prices. To avoid this contradiction, we state that the value of a gram of gold has diminished, a phenomenon for which a cause may be found and which explains the general movement of prices while respecting the principle of Causality. But if we examine the meaning of the statement that the value of the gram of gold has diminished, we perceive that reference is being made to some ideal unit of value. This ideal unit can only be defined by affirming that it is the one which gives us expressions of phenomena seeming to verify the principle of Causality.

Thus, the ideal unit of value is defined like the ideal unit of length, like the ideal unit of time and like all such units. As for practical units, they are never more than provisional, approaching more or less closely the ideal ones. Thus, the meter defined as a forty-millionth part of the terrestrial meridian is not so good a unit as the standard platinum meter

just as the silver monetary standard is not so good a unit as the gold monetary standard. In order to compare economic phenomena of two different periods, it is first necessary to express them in terms of a unit of price belonging to the system of an ideal unit. This is what is done in practice, account being taken of the coefficient of the general increase in prices which is admitted to be the inverse of the coefficient of depreciation of the monetary standard with respect to the ideal unit of value.

We see that here, as in other sciences, our rôle is not confined to taking passive note of the appearances revealed to us by our senses. The choice of our unit of measure shapes the expression of empirical laws and the unit is chosen so that the expression of those laws shall conform to the principle of Causality. This principle is, therefore, a garb which we have placed upon the world, but its imposition was possible, and that was not evident.

Armed with this unit of value, we observe the economic phenomena from which we derive the laws which constitute the empirical branch of political economy. These laws are "economic reality" just as the laws of Kepler, for example, are astronomic reality. They express the common characteristic of respective groups of appearances. Thus, observing all the sales of wheat which take place in a given place at a given moment, we observe that a hectoliter of wheat is exchanged for 75 francs. We say,

“Wheat is worth 75 francs per bushel.” In the same way, we enunciate the following laws:

“Buyers try to buy at the lowest prices. Sellers try to sell at the highest prices.

“When, in the same market (at a given price), the demand is greater than the supply, prices go up; when the supply is greater than the demand, prices go down.

“In a given market at a given moment, a single price prevails.

“This price is a stable price in the sense that, if for some reason one deviates from it, one is naturally brought back to it, etc. . . .”

We do not propose to state all of the laws which it is possible to discover. That would be the work of a treatise on empirical political economy. We wish only to study their characteristics.

First of all, their existence does not seem susceptible of doubt. Every time we study the economic phenomena taking place in societies made up of a large number of individuals, we observe that the same antecedents always produce the same consequences. The scarcity of a given product on the market or the discovery of a useful attribute of such a product has always led to a rise in its price. The issue of an inconvertible currency has always led to a depreciation of the monetary standard of a country with respect to that of other countries. These laws, as well as others we could cite, seem rigorous and immutable. If they do not always govern the succession of phenomena, it is not that they have ceased to be true, but that the conditions on which

their action depends have ceased to exist. We could repeat in this connection all that was said in Chapter XII, and that is applicable to all empirical laws.

The laws of political economy are statistical laws in the sense that the sequential relations which they enunciate result from the presence and simultaneous activity of a large number of isolated elements. They are thus quite comparable to the laws of gases, individuals playing in political economy the rôle of molecules in the kinetic theory. The future appearances which they make it possible to foresee can, therefore, never have more than a high degree of probability of occurrence and the probability of their coming to pass will be greater according as the antecedent conditions are more clearly and fully understood.

Thus, economic laws are more or less true according to circumstances. Those which are rigorous on the stock exchange are only approximated in a market where the goods are less well standardized, and where contacts among the participants and publicity of transactions are less assured. The smaller the number of traders the less perfectly will these laws be realized. There is no political economy of the individual any more than there is a thermodynamics of the molecule.

The great difficulty in observing these laws is that we are ourselves one of the elements combining to bring about the appearance whose advent they foretell. We blend the subjective notions of which we are aware with the phenomena which we observe and we

want to connect average properties with those of an isolated element. In seeking these laws, we are in the position of a molecule that wished to comprehend the properties of gases.

In order to discover the empirical laws of political economy, we must force ourselves to consider society as a whole and us as no part of it; to look only for sequential relations pure and simple and not the ways of their realization as that is the task of psychology.

Furthermore, the equilibrium resulting from the action of these numerous elements may take a long time in establishing itself, just as that brought about by the diffusion of gases may, so that the play of economic laws may at times be veiled by passive obstacles which delay it. Speculators on the market play the same rôle as catalysts in chemistry. By buying when there is a tendency toward a rise in prices, they hasten the change in the price. This brings the market into that state of equilibrium corresponding to the new conditions. They thus prevent the formation of false equilibria.

Finally, economic laws are by their very nature particularly difficult to discover. Experimentation is almost impracticable or, at least, very difficult because of the impossibility of isolating the antecedents. As in the study of biological laws, where the same conditions exist although to a lesser degree, one is driven to prolonged observation which makes it possible to profit by accidental variations of the



antecedents and by the study of economic cataclysms. The latter, like pathological cases, are particularly interesting because of the rapidity of the variations which they produce.

However, great as these difficulties are, they have not prevented us from discovering a certain number of laws, some of which were stated above and may be regarded as finally settled.

These laws express nothing more than sequential relations empirically observed. Now we wish to make of them logical laws, *i. e.*, expressions of causal relations, deducible from a knowledge of ultimate reality, consistent with each other and constituting the nature of things.

As this ultimate reality is not given, we create it by enunciating a certain number of axioms and definitions which we make the premises of reasoning for deducing the empirical laws. Extending the theory thus constructed, we derive therefrom new laws which observation may either confirm or invalidate. In the first case, the logical theory remains true. We say that the created causes are also true and that the theory derived from them is a Euclidean theory. When the coincidence of the consequences with observed laws does not take place, the theory remains logical but it is no longer true. We say that it is a non-Euclidean theory.

Before undertaking the study of economic theories, we must first answer an objection which always presents itself to the mind of the lay sceptic. Polit-



ical economy, he claims, is not in the same category with the mathematical sciences because too many factors determine its phenomena, because we do not know them all and cannot translate them into numbers, and finally and especially, because the free will of individuals exercises an influence which we cannot foresee.

These objections do not seem to us to be well grounded. The sole question is whether there are economic laws, not what factors determine them. Now the existence of these laws seems certain. The interpretation of statistics, which are to political economy what astronomical observations are to celestial mechanics, permits the enunciation of laws which cannot be doubted. What matters then the rôle of free will in human life? These laws once known, we do not concern ourselves with discovering their causes, a problem devoid of meaning, but only with creating them. And when certain causes permit us to discover by the deductive route laws empirically discovered, those causes are good, whatever ideas one may have about the physical or moral nature of individuals, free or otherwise.

If, under certain conditions, economic laws cease to be verified, an attempt is made to retain them by imagining accessory phenomena due to disturbing causes chosen for this purpose. The absence of competition and the lack of publicity in exchanges thus play in political economy a rôle similar to that of

friction in mechanics and they make it possible to represent by a single symbol a multitude of factors of which we are ignorant.

#### A.—THE EUCLIDEAN THEORIES OF POLITICAL ECONOMY

We do not propose to set out a complete rational theory of political economy. We wish only to show that the construction of such a theory is possible and to study its nature.

For this, we shall enunciate principles which may serve as its bases and we shall show that the initial theorems derived from them coincide with corresponding laws empirically discovered. Classical political economy already displays the character of a rational theory in many of its parts and under its usual form.

Having defined the terms and enunciated the axiom of personal interest, *viz.*, “Man constantly seeks what he believes to be the greatest satisfaction of his wants and those of his family by the means which he thinks call for the least effort,”<sup>1</sup> we may rediscover, by the purely deductive route and in a rigorously scientific manner, the law of supply and demand, of stability of prices, and the theory of monopoly prices.

Now the deductive powers of man are much more limited under the syllogistic form than under the analytical form. The lines of reasoning, even when they are accessible, are long and difficult, and the

<sup>1</sup> C. Colson, *Cours d'Economie Politique*, Vol. I.

complexity of the process prevents us from indefinitely bringing together the newly acquired results and the axioms or the propositions previously deduced. For this reason the process of deduction breaks down before we have deduced all the appearances of economic life.

On the other hand, it is possible by defining a certain number of symbols and positing only two axioms to rediscover in a simple and rigorous manner all the empirical laws already known. Mathematical political economy is thus built up. It has as an advantage over ordinary political economy, in addition to the precision of its form, the fact that it can be developed indefinitely.

We are going to sketch such a theory. We have borrowed some of the initial propositions from the "*Traité d'économie politique pure*" of Leon Walras although the point of view which that author adopts is entirely different from ours. We shall be guided in the definition of terms by a mechanical analogy.

The fundamental fact which gives rise to rational mechanics is the existence of movements which its purpose is to explain. Similarly, the fundamental fact of political economy is the existence of exchanges whose laws its purpose is to rediscover.

Perceiving that the cause of movements which we produce is the effort which we exert on bodies, we lay down the principle that all movements of material bodies are the manifestation of an effort which

is applied to them, to which effort we give the name of force. Similarly we are conscious of never effecting an exchange except when we feel a want for the object we acquire. We lay down, then, the principle that an exchange is only a manifestation of the different wants of the individuals effecting it.

The notion of force having been introduced, we observe that the variation in the speed of a body upon which we exert an effort is greater in proportion as the effort seems greater to us or the mass of the body smaller. We thus conventionally define *force* as a magnitude measured by the number

$$F = m \frac{dv}{dt}$$

Similarly, if after buying a certain quantity of a commodity at a certain price we buy less at a higher price, we observe that we cut down less on the quantity bought, the greater our need of the commodity. Under these conditions,  $q$  being the quantity bought at the price,  $p$ , we define conventionally our want or need of a commodity by the equation

$$n = - \frac{dq}{dp}$$

The first objection to which this definition gives rise is that it represents by a number something which is not measurable. It would thus seem to be contradictory and, hence, illegitimate.

It is, as a matter of fact, impossible to prove that the number  $n = - \frac{dq}{dp}$  actually measures the want

we have of an object, and we think we can even affirm that the want revealed to us by consciousness is a magnitude which is not measurable, the two wants which we say are of different intensity being really two states of consciousness qualitatively different, for which there is no common measure. On the other hand, the numbers obtained by the preceding definition for two different articles are of the same nature and are measured by means of a common unit; they are, therefore, susceptible of equality and of addition, all of which, from the psychological point of view, is absolutely false.

The quantity  $n = -\frac{dq}{dp}$  is, therefore, not the measure of a state of consciousness. It is an entirely arbitrary symbol created with reference to a state revealed to us by consciousness and resembling it as much as a number can resemble a feeling. The justification of the definition which created it can only be found in the coincidence of the consequences which can be derived from it with empirical laws.<sup>1</sup>

<sup>1</sup> It will be seen that, if we regard  $dp$  and  $dq$  as percentage increments, the symbol chosen by the author to represent the intensity of a need or want is also the expression for the elasticity of demand. The inelasticity of demand is assumed to be directly proportional to the intensity of the want. The appropriateness of this symbol for measuring a want is open to question, although it must be remembered that the author is not seeking qualitative similarity. The expression is always negative, increasing as the intensity of want increases, from  $-\infty$  to 0. It does not seem consonant with the feeling to express the most intense want of which one is capable by 0. An expression would more resemble the feeling which should

This remark concerning the arbitrary character of our definition can in no wise diminish the value of the edifice built upon it. The vector, force, which is at the base of mechanics is as different from an effort as the number  $n$  is from a need. The effort we exert upon a body in order to move it is not a measurable magnitude any more than a need is. It depends upon the form of the body and upon the way in which we grasp it. The efforts exerted upon two different bodies are not of differing intensity, they are qualitatively different and have no common measure. We represent them by forces, we speak of their equality and of their addition and yet the value of mechanics has never been subjected to doubt for this reason.

This difference between effort and force confirms our point of view in other respects. The definitions which are at the base of the sciences do not pretend, as is commonly thought, to represent what is defined. They create what is defined to serve as a premise for a process of reasoning which will rediscover the phenomena of experience. They create it for this purpose only but insist on creating it in the image of the only causes actually known to us, namely, those revealed to us by consciousness.

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start with 0 for no consciousness of want and increase as the want increases. It is significant that a few pages later when the author has occasion to make use of a symbol for expressing the intensity of a want, he finds this one to break down and makes use of another, namely,  $n = f(q)$ .—Ed.



It is, therefore, perfectly legitimate to give a mathematical definition of want, without trying to find out whether that definition really represents the need we have of an object. It is, moreover, sufficient, in order to eliminate the objection, to name by the letter  $n$  the quantity it represents and to admit that it is different from the need of which we have subjective knowledge.

The definition will be good if, joined to certain axioms, it enables us to construct a theory by the aid of which we may deduce the phenomena of experience. We shall see that this is really the case.

We shall lay down the two following axioms.

*Axiom I.*—The want we have of a commodity decreases when the quantity of it which we possess increases. It starts out with a certain value above zero for a zero quantity possessed, and reaches the value zero for a finite quantity (satiety).

If we represent as the *abscissa* the quantity possessed and the want as the *ordinate*, we can represent Axiom I by the accompanying curve.

The length of  $O\beta$  thus measures the want we have of a given commodity  $A$  when we possess none of it. The length  $Oq$  measures the quantity of the commodity which satisfies the totality of our want of it. When we possess of  $A$  a quantity above  $Oq$ , we no longer want it. All the quantities possessed beyond  $Oq$  do not answer any want. Analytically, the desire for commodity  $A$  is a certain function of the quantity of it possessed.



We shall represent by  $n$  the need which a certain individual feels for the commodity  $A$ , and by  $q$  the quantity of it which he possesses. We can then put down

$$n = f(q)$$

$f$  being an unknown function, but fully determined for a given individual at a given moment. This function may vary from individual to individual, but,

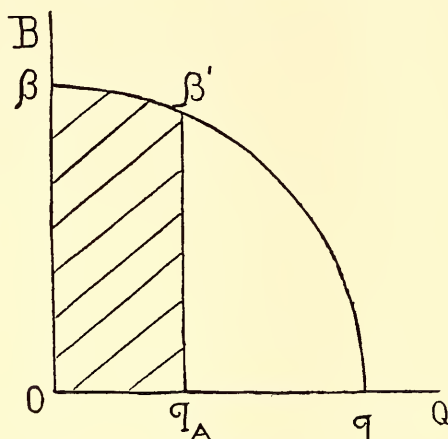


FIG. 1.

by virtue of Axiom I, it always decreases when  $q$  increases.

We can further observe, in this connection, that we proceed the same way in mechanics, defining the resistance of friction as an unknown function of the velocity, but increasing with it.

This having been laid down, we shall call the quantity  $U$ , defined by the expression

$$U = \int_0^{q_a} \beta dq = \int_0^{q_a} f(q) dq,$$

the utility of the quantity  $q_a$  of the commodity  $A$  for the individual considered.

This quantity is evidently that measured by the shaded area in figure 1. We see at once that  $U$  is an integral of the function  $f(q)$  and we may write  $U = F(q)$ . As a consequence of this definition, we observe that, if an individual possesses increasing quantities of  $A$ , then each time an infinitesimal increment,  $dq$ , is added, the increase in utility resulting from such increase in quantity is smaller, the larger the quantity of  $A$  already possessed. The supplement of utility is in each case equal to the product of the quantity of  $A$  added and the magnitude of the need ( $n$ ) which it satisfies.

In order to complete our system of causes, it now remains to enunciate the following axiom:

*Axiom II.*—Each individual possesses goods in a finite quantity, and seeks, by the exchange which he makes, to acquire various goods in quantities such that the sum of the respective utilities of the goods which he possesses shall be maximum.

Now, we saw that the price of an object is the quantity of money which it is necessary to give in order to procure that object through exchange.

In a more general manner, we shall call the price of an object,  $A$ , in terms of an object,  $B$  (and we shall designate it by  $p^a/b$ ) the quantity of  $B$  required in exchange for the unit quantity of  $A$ . From this it follows that, in order to acquire  $q_a$  of  $A$ , it is necessary to give  $q_a p^a/b$  of  $B$ , and, inversely, in exchange

for  $q_a$  of  $A$ , one may obtain  $q_a p^a/b$  of  $B$ . It is immediately seen that we have

$$p^a/b \cdot p^b/a = 1.$$

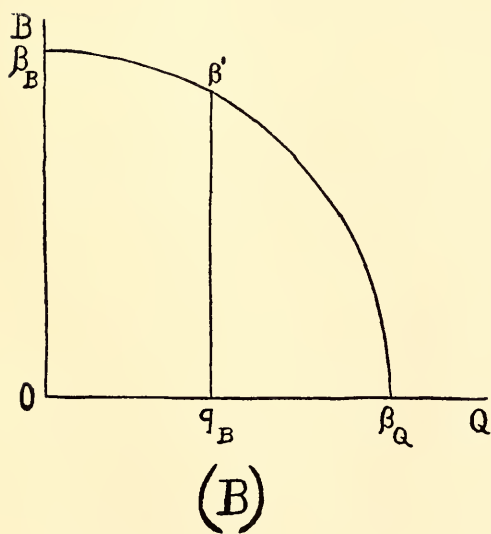
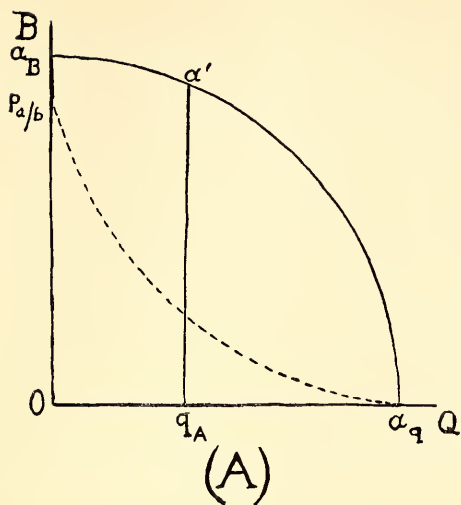
In order to construct a general theory of exchange, we shall begin by studying a simple case. This study, though not strictly indispensable to our exposition, will accustom the reader to the notations employed. An individual possesses a quantity  $Q_b$  of the commodity  $B$  and none of  $A$ . Furthermore, his wants of  $A$  and  $B$  are defined as functions of the quantities possessed, by the curves  $A$  and  $B$ , or by the equations

$$n_a = f_a(q_a) \qquad n_b = f_b(q_b)$$

$n_a$  and  $n_b$  being the wants of the individual under consideration for the commodities,  $A$  and  $B$ ;  $q_a$  and  $q_b$ , being the quantities thereof which he possesses, and  $f_a$  and  $f_b$  being functions fully determined for the moment under consideration. We shall suppose that he does not feel the need for any commodities other than  $A$  and  $B$ .

He goes into the market where the price of  $A$  with respect to  $B$  is  $p^a/b$ . By virtue of Axiom II, he is going to exchange a part of the  $B$  which he has for a certain quantity of  $A$  so as to render maximum the sum of the utilities afforded to him by the quantities of  $B$  and  $A$  which he holds after the exchange.

Let  $q_a$  be the quantity of  $A$  he is going to ask for. At the price  $p^a/b$ , he will have to give in exchange a quantity of  $B$  equal to  $q_a p^a/b$ .



*Theorem.*—For each price of  $A$  in terms of  $B$ , the demand for  $A$  by our purchaser will be completely determined. Otherwise expressed  $d_a = \phi(p^a/b)$ . As shown above, our purchaser will have to possess after the exchange, a quantity  $q_a$  of  $A$  so that the sum Area  $0\beta_b\beta'q_b$  (Fig. B) + Area  $0\alpha^b\alpha'q_a$  (Fig. A, p. 117) shall be maximum, that is, so that the expression,

$$(1) \quad \int_0^{q_a} f_a(q_a) dq_a + \int_0^{q_b} f_b(q_b) dq_b$$

shall be maximum.

Now in order to have 1 unit of  $A$ ,  $p^a/b$  of  $B$  must be given. Therefore, in order to have  $q_a$  units of  $A$ ,  $p^a/b$  of  $B$  must be given. It follows from this that after the exchange, when our purchaser will possess  $q_a$  of  $A$ , he will have only  $q_b$  of  $B$ , shown by the expression

$$q_b = Q_b - q_a p^a/b$$

By replacing in equation (1)  $q_b$  with this value, we obtain the expression in terms of the single variable  $q_a$ ; (that is, observing that  $dq_b = -p^a/b dq_a$ , we have—ED.)

$$(2) \quad \int_0^{q_a} f_a(q_a) dq_a - \int_0^{Q_b - q_a p^a/b} f_b(Q_b - q_a p^a/b) p^a/b dq_a$$

which must be maximum.

Its derivative with respect to  $q_a$  must be zero, which gives us the condition

$$(3) \quad f_a(q_a) - f_b(Q_b - q_a p^a/b) p^a/b = 0, \\ \text{(or } f_a(q_a) - p^a/b f_b(q_b) = 0 \text{—ED.)}$$

This expression solved with respect to  $q_a$  proves that we have

$$q_a = \phi(p^a/b)$$

which gives, for each price of  $A$  in terms of  $B$ , the quantity of  $A$  which our purchaser had to demand in order to render maximum the total utility of the goods he possessed. We have, in consequence,

$$d_a = \phi(p^a/b)$$

which was to be proved.

This laid down, formula (3) shows us that the total utility will be maximum when we have

$$\frac{f_a(q_a)}{f_b(q_b)} = p^a/b$$

But  $f_a(q_a)$  is the measure of the need  $n_a$  of our purchaser for commodity  $A$  when he possesses of it the quantity  $q_a$ . Similarly,  $f_b(q_b)$  for  $B$ . Hence this theorem:

*Theorem.*—When an individual possesses commodities  $A$  and  $B$  in the quantities  $q_a$  and  $q_b$ , the total utility resulting from the possession of these commodities is at maximum when the ratio of the wants for  $A$  and  $B$  that remain to be satisfied is equal to the price of  $A$  with respect to  $B$ . (Theorem enunciated by L. Walras) or otherwise stated, when we have

$$(4) \quad \frac{n_a}{n_b} = \frac{f_a(q_a)}{f_b(q_b)} = p^a/b$$

We will now show that these same results apply to the general case where an exchange of several com-

modities is made by several individuals by means of money. We shall employ the same notation.

Money is a conventionally chosen commodity with respect to which the prices of all other commodities are determined. By definition the price of the commodity, money, is equal to unity.

The definition of the magnitude of a want  $\left(-\frac{dq}{dp}\right)$  does not seem to apply to money, the price of which is constant. To obviate this difficulty we might state the price of this commodity in terms of the ideal unit of value previously defined. However, to simplify matters let us extend to money the principle in accordance with which a want felt by an individual was defined as a function,  $f_a(q_a)$ , of the quantity  $q_a$  of money possessed, and that Axiom I gives the meaning of its variations. We can, therefore, write:

$$n_a = f_a(q_a)$$

This laid down, let us consider the case of an individual possessing quantities  $Q_a, Q_b, Q_c \dots$  of the commodities  $A, B$ , and  $C$ ,  $A$  being money. The need of each which he experiences is defined by the functions:

$$n_a = f_a(q_a) \quad n_b = f_b(q_b) \quad n_c = f_c(q_c)$$

In the market to which he goes, the commodities  $A, B, C$ , have the respective prices, 1,  $p_b, p_c \dots$ . Under these conditions our individual is going to engage in exchanges so as to possess the commodities  $A, B, C, \dots$  in quantities  $q_a, q_b, q_c$  affording him the maximum utility.





evidently be determined by equation (6). Hence this theorem:

*Theorem.*—For every price of  $B, C, D$ , in terms of  $A$ , the quantity of  $A, B, C, \dots$  offered or asked for by each individual present on the market has a fully determined value.

This posited, let us solve system (7) with respect to  $p_b, p_c, \dots$  We have:

$$p_b = \frac{f_b(q_b)}{f_a(q_a)} \quad p_c = \frac{f_c(q_c)}{f_a(q_a)} \dots \text{etc.}$$

(Since

$$f_a(k - q_b p_b - q_c p_c \dots) = f_a(q_a) \text{—ED.})$$

Now  $f_b(q_b)$  is the magnitude of the want felt by our individual for  $B$ , after the exchange; in other words, it is his want for  $B$  still remaining to be satisfied. Hence this theorem:

*Theorem.*—When an individual possesses commodities  $B$  and  $A$  in quantities such that the utility resulting from their possession is maximum, the ratio of the needs of  $B$  and  $A$  remaining to be satisfied is equal to the price of  $B$  with respect to  $A$

$$p^{b/a} = \frac{f_b(q_b)}{f_a(q_a)}.$$

### *Variations of Price*

In order to study the variations of price in a market, let us call the total want of  $A, B, C, D, \dots$  in the market, the sum of the respective wants felt by the various individuals for these commodities. We shall

obtain, designating by the capitals the quantities thus obtained,

$$N_a = F_a(Q_a)$$

$$N_b = F_b(Q_b)$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

and since for each individual

$$\frac{f_b(q_b)}{f_a(q_a)} = p_b$$

we have further

$$(8) \quad \frac{F_b(Q_b)}{F_a(Q_a)} = p_b$$

(Denote by primes, seconds, etc., the wants of the several individuals. Then we have

$$\frac{f_b(q_b)}{f_a(q_a)} = p_b, \quad \frac{f'_b(q'_b)}{f'_a(q'_a)} = p_b, \quad \frac{f''_b(q''_b)}{f''_a(q''_a)} = p_b, \text{ etc.}$$

$$\therefore \frac{f_b(q_b)}{f_a(q_a)} = \frac{f'_b(q'_b)}{f'_a(q'_a)} = \frac{f''_b(q''_b)}{f''_a(q''_a)} = \dots$$

$$(8) \therefore \frac{f_b(q_b) + f'_b(q'_b) + f''_b(q''_b) + \dots}{f_a(q_a) + f'_a(q'_a) + f''_a(q''_a) + \dots} = \frac{F_b(Q_b)}{F_a(Q_a)} = \frac{f_b(q_b)}{f_a(q_a)} = p_b \text{ —ED.)}$$

This posited, we see that, if the total quantity  $Q_a$  of money on the market increases,  $F_a(Q_a)$  decreases, by virtue of Axiom I. In consequence, other conditions remaining the same, the prices  $p_b, p_c, \dots$ , of all the commodities on the market go up. We thus rediscover an empirical law which was found verified in the United States during and after the war. The influx of gold was alone sufficient to cause an increase of prices.

If the total quantity  $Q_b$  of  $B$  on the market is suddenly diminished,  $F_b(Q_b)$  increases. The price of  $B$  increases. This is another empirical law which it has been possible to verify upon many occasions (fire, shipwreck, levy, bad crops, etc.).

It may happen that, the quantity of  $B$  on the market remaining constant, the desires of the different individuals for  $B$  increases simultaneously due, for example, to good publicity or the discovery of some new property of the product  $B$ . In that case  $F_b(Q_b)$  increases and the price of  $B$  increases.

On the other hand, the need of the individuals for money may increase as the result of opportunities for advantageous investment, for example. In that case  $F_a(Q_a)$  increases and prices decrease.

We have thus rediscovered a certain number of empirical laws relative to variations of prices. But we have seen especially that the expression, value of a commodity, had no meaning.<sup>1</sup> Price alone has meaning, and the preceding discussion has shown us upon what factors it depends.

<sup>1</sup> "Mais nous avons vu surtout que l'expression, valeur d'une marchandise, n'avait aucun sens." The author apparently means that value in the sense of a necessary and invariable attribute is without meaning. Value in the sense of a ratio expressing the terms on which one commodity exchanges with another has a very important meaning, indeed it is one of the most fundamental terms in political economy. As ordinarily used in English, "price" is merely the "value" of a commodity expressed in terms of money. It will be noted that in almost the next paragraph the author himself uses the expression, "The value of the products bought."—Ed.

*Variations of Supply and Demand as Functions  
of Price*

Let us return to our trader possessing upon his arrival in the market the quantities  $Q_a, Q_b, Q_c \dots$  of  $A, B, C \dots$ . Let us put

$$q_a = Q_a + D_a \quad (\because dq_a = dD_a)$$

$$q_b = Q_b + D_b \quad (\because dq_b = dD_b)$$

$$q_c = Q_c + D_c \quad (\because dq_c = dD_c)$$

(since  $Q_a$  is constant—ED.)

$q_a, q_b, q_c \dots$  being the quantities of  $A, B, C \dots$  that he will have to possess after the exchange in order for the utility of the goods he holds to be maximum.

There is effective demand for  $A, B, C$ , by our trader if  $D_a, D_b, D_c \dots$  are positive. There will be an offering by him if they are negative.

The value of the products bought being necessarily equal to that of the products sold, we have always

$$(9) \quad D_a + D_b p_b + D_c p_c \dots = 0$$

or

$$D_a = -D_b p_b - D_c p_c \dots$$

By replacing in the equation (5)  $q_a$  by  $Q_a + D_a$  and  $D_a$  by the above value, we obtain

$$\begin{aligned} & \int_0^{q_a} f_a(Q_a - D_b p_b - D_c p_c \dots) d(Q_a - D_b p_b - D_c p_c \dots) \\ & + \int_0^{q_b} f_b(Q_b + D_b) dD_b + \int_0^{q_c} f_c(Q_c + D_c) dD_c + \dots \end{aligned}$$

and system (7) is written:

$$(10) \quad \begin{cases} -f_a(Q_a - D_b p_b - D_c p_c \dots) \\ \quad \quad \quad p_b + f_b(Q_b + D_b) = 0 \\ -f_a(Q_a - D_b p_b - D_c p_c \dots) \\ \quad \quad \quad p_c + f_c(Q_c + D_c) = 0 \\ \dots\dots\dots \\ \dots\dots\dots \end{cases}$$

These equations define  $D_b, D_c \dots$  for each value of  $p_b, p_c \dots$

In order to study the variation of  $D_b$  as a function of  $p_b$ , let us differentiate the first equation (10).<sup>1</sup>

<sup>1</sup>The expression  $-f_a(Q_a - D_b p_b - D_c p_c \dots)p_b + f_b(Q_b + D_b) = 0$  is an implicit function of  $D_b$  and  $p_b$ . We may substitute  $u$  for 0 and

differentiate by the convenient formula  $\frac{dy}{dx} = -\frac{\frac{\delta u}{\delta x}}{\frac{\delta u}{\delta y}}$  where  $y = D\phi$

$$\frac{dD_b}{dp_b} = - \frac{-f_a(Q_a - D_b p_b - D_c p_c) + p_b D_b \frac{\delta f_a(Q_a - D_b p_b - D_c p_c \dots)}{\delta(Q_a - D_b p_b - D_c p_c \dots)}}{p_b \frac{2\delta f_a(Q_a - D_b p_b \dots)}{\delta(Q_a - D_b p_b \dots)} + \frac{\delta f_b(Q_b + D_b)}{\delta(Q_b + D_b)}}$$

In differentiating such an expression as  $f_b(Q_b + D_b)$  with respect to  $D_b$  we may use the formula  $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$  regarding  $y = f(v)$  and  $v = Q_b + D_b$  and  $x = D_b$ . It should be noted that such expressions as  $(Q_a - D_b p_b \dots)$ ,  $(Q_b + D_b)$ , etc., may be regarded as expressions denoting quantity. The  $f$  function of such an expression is by definition the need or want still remaining when the individual possesses that quantity of the commodity in question. But the derivative of the want with respect to the quantity  $\left(\frac{dn}{dq}\right)$  is always negative. Hence all such expressions as

$$\frac{\delta f_a(Q_a - D_b p_b \dots)}{\delta(Q_a - D_b p_b \dots)}$$

We obtain:

$$\left[ -f_a(Q_a - D_b p_b - D_c p_c \dots) + p_b D_b \frac{\partial f_a(Q_a - D_b p_b \dots)}{\partial (Q_a - D_b p_b \dots)} \right] dp_b \\ + \left[ \frac{\partial f_a(Q_a - D_b p_b - D_c p_c \dots)}{\partial (Q_a - D_b p_b \dots)} p_b^2 + \frac{\partial f_b(Q_b + D_b)}{\partial (Q_b + D_b)} \right] dD_b = 0$$

That is

$$(11) \quad \frac{dD_b}{dp_b} = - \frac{-f_a(Q_a - D_b p_b \dots) + p_b D_b \frac{\partial f_a(Q_a - D_b p_b \dots)}{\partial (Q_a - D_b p_b \dots)}}{p_b^2 \frac{\partial f_a(Q_a - D_b p_b \dots)}{\partial (Q_a - D_b p_b \dots)} + \frac{\partial f_a(Q_b + D_b)}{\partial (Q_b + D_b)}}$$

By virtue of Axiom I, the denominator is always negative or zero. The derivative, therefore, has the sign of

$$-f_a(Q_a - D_b p_b \dots) + p_b D_b \frac{\partial f_a(Q_a - D_b p_b \dots)}{\partial (Q_a + D_b p_b \dots)}$$

When  $D_b$  is positive, which is the case when there is an effective demand, the derivative  $\frac{dD_b}{dp_b}$  is always negative or zero,  $f_a(Q_a - D_b p_b \dots)$  being always positive and  $\frac{\partial f_a(Q_a - D_b p_b \dots)}{\partial (Q_a - D_b p_b \dots)}$  always negative, following Axiom I. Therefore, demand decreases or remains constant when the price increases.

When the price is zero, the corresponding demand is determined by the equation

$$f_b(Q_b + D_b) = 0$$

(substituting 0 for  $p_b$  in equation 10—ED.)

whence we obtain

$$D_b = (\text{a finite quantity—ED.}), \text{ say, } b$$

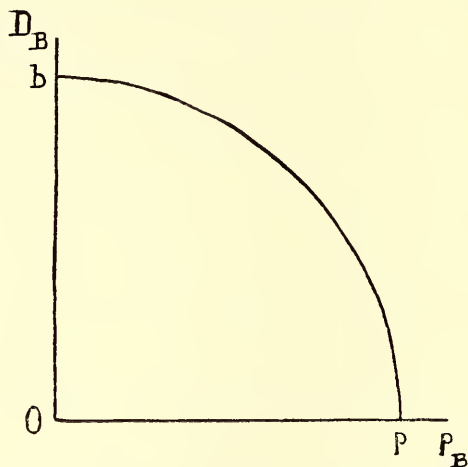


(which is the numerical value corresponding to satiety).

Furthermore, in order for the demand to be zero, we must have (substituting 0 for  $D_b$  in equation 10—ED.)

$$(12) \quad p_b = \frac{f_b(Q_b)}{f_a(Q_a - D_c p_c \dots)} \\ = (\text{a finite quantity—ED.}), \text{ say, } p.$$

from which we obtain the theorem:



*Theorem.*—When the price increases from zero to a finite value,  $p$ , the demand decreases from a finite value  $b$  to zero. We may represent its variations by the accompanying curve. (Note that in this figure prices are represented as abscissas, and quantities (demands) as ordinates.—ED.)

Let us suppose now that  $D_b$  is negative, in other words, that our trader has to offer for sale some

of  $B$  in order to arrive at the maximum satisfaction. Let us put down  $-D_b = 0_b$ . All the calculation having been made, we obtain as before:

$$(13) \quad \frac{d0_b}{dp_b} = + \frac{+f_a(Q_a + 0_b p_b + \dots) + 0_b p_b \frac{\partial f_a(Q_a + 0_b p_b + \dots)}{\partial (Q_a + 0_b p_b + \dots)}}{-\frac{\partial f_a(Q_a + 0_b p_b + \dots)}{\partial (Q_a + 0_b p_b + \dots)} p^2_b - \frac{\partial f_a(Q_b - 0_b)}{\partial (Q_b - 0_b)}}$$

The denominator is always positive. The derivative, therefore, has the sign of

$$(14) \quad f_a(Q_a + 0_b p_b + \dots) + 0_b p_b \frac{\partial f_a(Q_a + 0_b p_b + \dots)}{\partial (Q_a + 0_b p_b + \dots)}$$

We must study the variations of  $0_b$  when  $p_b$  increases from the value  $p$  previously found. When  $p_b = p$ ,  $0_b = D_b = 0$ . When  $p_b$  increases from  $p$  to

$$(15) \quad p' = - \frac{f_a(Q_a + 0_b p_b + \dots)}{0_b \frac{\partial f_a(Q_a + 0_b p_b + \dots)}{\partial (Q_a + 0_b p_b + \dots)}}$$

the derivative is positive. For the value,  $p'$ , it becomes null and becomes negative for all the values of  $p$  greater than  $p'$ . Thus supply passes through a maximum for  $p = p'$ ; let  $\pi$  be its value.

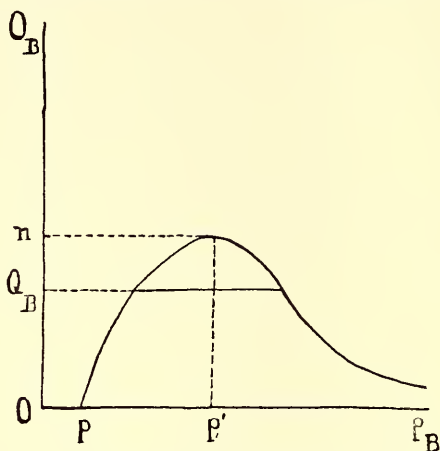
When  $p_b$  is infinite, the equation (9)<sup>1</sup> shows that we necessarily have  $0_b = 0$ . Hence this theorem:

*Theorem.*—When the price increases from a value  $p$  to an infinite value, the offering by our seller of  $B$  increases from zero to a value  $\pi$ , which it attains for

<sup>1</sup> The equation is  $D_a + D_b p_b + D_c p_c \dots = 0$  or  $D_a - D_b p_b + D_c p_c \dots = 0$ . From this we have  $0_b p_b = D_a + D_c p_c \dots$  or  $0_b = \frac{D_a + D_c p_c \dots}{p_b}$ . Hence if the numerator is finite, when  $p_a = \infty$ , we have  $0_b = 0$ .—ED.

a price  $p'$  and then decreases to zero which it reaches for an infinite price. Hence the accompanying curve.

*Remarks:* It may happen that, for certain values of the price, the supply found is greater than the quantity of merchandise  $Q_b$  possessed. In that case, we evidently put down  $0_b = Q_b$  and the entire part of



the curve of ordinate above  $Q_b$  is replaced by a line, of ordinate  $Q_b$ , parallel to the axis of prices.<sup>1</sup>

<sup>1</sup> The author's reasoning that the supply curve has one or more maximum points is not conclusive. It is based on inferences derived from equations (9) and (14). (a) Equation (9) after substituting  $-0_b$  for  $D_b$  may be reduced to the form  $0_b = \frac{D_a + D_c p_c}{p_b}$  (See footnote 5). Hence when  $p_b$  is infinite,  $0_b$  will equal zero, if the numerator is finite. But the numerator may be infinite, i. e.,  $0_b = \frac{\infty}{\infty}$ . This supposition would permit  $0_b$  to be finite, and if  $0_b$  is finite when  $p_b$  is infinite not only may the numerator be infinite, but it must be infinite, since an infinite price for  $p_b$  implies an infinite demand for money, if there are any sales. (b) The author shows

*Determination of Price on the Market. Law of  
Supply and Demand*

Let us consider a market upon which a certain number of traders are present.

*Theorem.*—The buyers are seeking to buy at the lowest possible price.

In fact, according to the axiom of maximum utility, the trader who wishes to buy a certain quantity of  $B$ , will look for a price which will leave in his posses-

that the derivative  $\frac{d0_b}{dp_b}$  takes the sign of the expression (14), and is positive, indicating an upward slope of  $0_b$  as  $p_b$  increases until a value  $p' = \frac{f_a(Q_a + 0_b p_b \dots)}{0_b \frac{\delta f_a(Q_a + 0_b p_b \dots)}{\delta(Q_a + 0_b p_b \dots)}}$  is reached from which point it de-

clines to 0. We may grant that  $0_b$  may increase to the value  $p'$  (though it may not if the entire quantity  $Q_b$  is exhausted before that point is reached). But the only evidence given of a subsequent decline is that  $p_b$  approaches  $\infty$ ,  $0_b$  must approach 0. But we have just shown that this is not necessarily true. The supply curve may ascend to an asymptote parallel to the axis of prices and distant from it by the amount  $0_b = p'$ . Under this supposition  $\frac{d0_b}{dp_a}$  would approach 0 as  $p_b$  approached  $p'$ , and would become 0 when  $p_b$  reached infinity. If the view here taken in regard to the supply for an individual is correct, the general supply curve for a market will also be a constantly ascending curve and will cut the demand curve (if it cuts it at all) in only one point and the subsequent discussion involving several possible points of intersection becomes superfluous.

It may interest the reader to note that the right hand member of

equation (13) may be written  $\frac{n_a + 0_b p_b \frac{\delta n_a}{\delta q_a}}{-p_b^2 \frac{\delta n_a}{\delta q_a} - \frac{\delta n_b}{\delta q_b}}$ , where  $n_a$  and  $n_b$

represent the need or want of money and commodity  $B$  respectively and  $q_a$  and  $q_b$  the quantity of money and of commodity  $B$  retained, when the sum of the utilities afforded by  $A$  and  $B$  is maximum.—Ed.

sion the greatest quantity of  $A$ , *i. e.*, he will look for the lowest price. We can show in the same way that the sellers are trying to sell at the highest possible price.

From these two theorems it follows that buyers and sellers will not strike a bargain unless they are convinced that they cannot strike a better one. Hence this theorem:

*Theorem.*—In order that two commodities may be exchanged, then at each moment in a given market there can be but one price for one with respect to the other.

This posited, let us suppose that the quantity of the commodity,  $B$ , offered at a certain price  $p_b$ , asked at random on the market, is less than the quantity demanded at that price. The buyers cannot all be satisfied. Now according to Axiom II, each of them seeks to conclude that bargain which will give him the maximum utility and, in consequence, attempts to obtain from sellers preference over the other buyers. The sellers, on the other hand, according to the preceding theorem, are trying to obtain the highest prices possible. In order to receive the preference, the buyers will, therefore, be impelled to offer to the sellers prices above that published on the market. The opposite phenomenon would have taken place if the supply had exceeded the demand.

The struggle which is thus set-up, spontaneously among the buyers in the first case, and among the sellers in the second case we call competition.

We now have at our disposal all the theorems necessary for the demonstration of the basic theorem of the theory of prices.

*Theorem.*—Competition which is spontaneously established on a market has the effect of establishing therein a stable price at which supply is equal to demand.

So long as this price is not reached, the price established in the market varies. It goes up when the



demand is greater than the supply and is lowered in the opposite case.

To prove this theorem, we shall call the total supply and demand upon the market the algebraic sum of the individual supplies and demands.

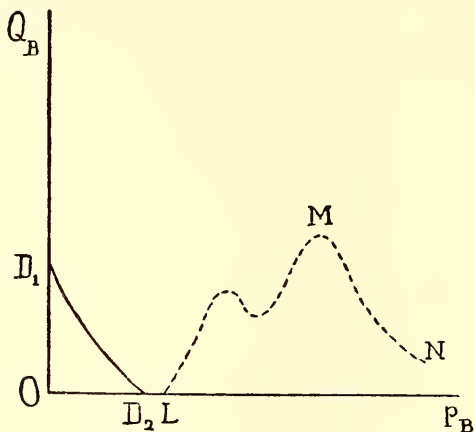
The curve representing the variations of the total demand as a function of the price is obtained by making for each price the sum of the ordinates of the curves of individual demands. It will always be decreasing like the individual curves.

It is immediately seen that the curve of the total supply, on the contrary, may not be as simple as the

individual curves, but may show several maxima and minima as the curve on page 133 does.

In order to demonstrate the preceding theorem, we must distinguish among several cases depending upon the relative positions of the curves of total supply and demand.

*1st Case.*—The curve of supply does not meet the curve of demand.



We see immediately that there is no price at which there are buyers and sellers at the same time. No exchange can take place. This is the case of so-called laboratory products whose net cost is so high as to make sales impossible except at prices at which there is no demand.

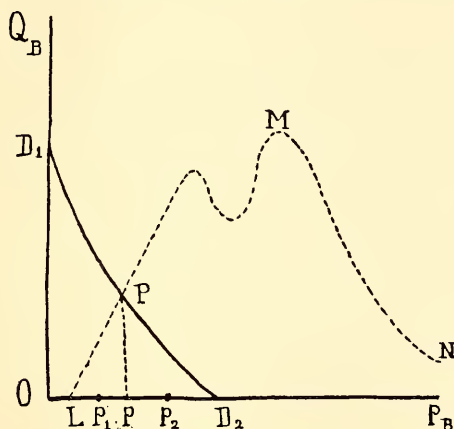
*2nd Case.*—The curve of demand meets the curve of supply at one point only.



The following reasoning applies to all cases of figures which come under one of the two following types:

Let  $P$  be the point of intersection of the abscissa  $O_p$  and let  $p$  be the price corresponding thereto.

Let us suppose that the first price offered at random on the market is  $p_1 < p$ . The accompanying figure shows us that, at this price, supply is less than demand. As a result of the competition which sets in

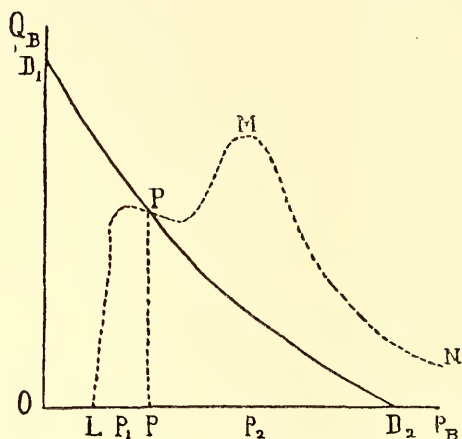


among the buyers, the latter are going to offer higher and higher prices. The rise in prices is not halted until all the buyers have been satisfied, that is, until the supply equals the demand. This takes place when the price  $p$  is reached.

It can be similarly proved that, if the first price offered had been  $p_2 > p$ , the supply would have been greater than the demand. Prices would have gone down to  $p$ .

When the price  $p$  is offered on the market all the buyers find sellers. Competition no longer exercising its influence, the price  $p$  does not vary. We say that  $p$  is the price of equilibrium on the market.

This price is a stable one because, if anyone deviates from it for some reason or other, competition sets in again, and the effect is to bring the price back to  $p$ .



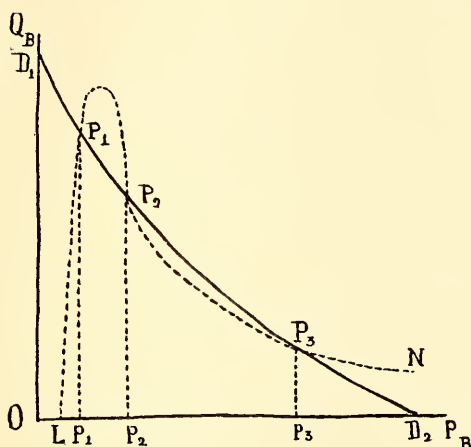
*3rd Case.*—The curve of supply meets the curve of demand at several points.

We can immediately observe that, the demand becoming nil for a finite value of the price and the supply only for an infinite value, the number of points of intersection is necessarily uneven. Let us suppose at first that the curve of supply meets the curve of demand at three points.

If the first price offered on the market is less than  $p_1$  or more than  $p_3$ , we can repeat precisely the same

demonstration as that in the paragraph above. The competition which sets in brings the prices back to  $p_1$  or  $p_3$  which are stable prices.

If the first price offered is such that  $p_1 < p < p_2$ , then supply is larger than demand, prices go down and the market price is brought back to the price of stable equilibrium  $p_1$ .



Similarly if  $p_2 < p < p_3$ , the price is brought back to the price of stable equilibrium,  $p_3$ .

If the first price offered is  $p = p_2$ , supply is equal to demand and the price does not vary. But if one deviates the least bit from this price, competition has the effect of bringing the exchanges made back to the prices  $p_1$  or  $p_3$ . The equilibrium which might be realized at the price  $p_2$  would be, therefore, an unstable equilibrium analogous to that of a cone placed

upon its apex, one which could not be maintained.

The theorem is, therefore, again proved in this case, which is distinguished from the second by the theoretical existence of two prices of stable equilibrium.

In general, furthermore, the offered price will be close to the lowest price  $p_1$  and equilibrium will be realized at that price.

If the curve of supply met the curve of demand at 5 points, we should again see the existence of a new theoretical price of unstable equilibrium and of a new price of stable equilibrium.

We have thus, by enunciating the preceding theorems, rediscovered by a purely deductive process the laws which regulate phenomena presented by some markets, stock exchanges for example. In conformity with the laws we have already demonstrated, we can affirm that the system of initial propositions which served as our premises, *i. e.*, the definitions of the magnitude of need, Axiom I, and Axiom II, are true for this market.

We can affirm that it represents the "nature of things" in this case and a little reflection will suffice to explain the meaning of this statement. It is quite evident that buyers and sellers on a stock exchange feel no need for the securities they exchange and that those securities have no utility for them in the subjective sense of the word. Only the profit they expect to obtain from their exchange has such utility. But this does not concern us at all. We wish to explain

the average appearances resulting from the conduct of each one of them and the need,  $n$ , which we create by a definition, explains them admirably. It constitutes, then, with the two axioms we have enunciated, the nature of things, the logical cause of the phenomena of stock-exchanges. The fact that consciousness does not reveal to us anything similar to the causes we have created does not in any way diminish their truth.

This posited, we observe the existence, alongside of stock exchanges, of other markets which the empirical laws enunciated no longer govern absolutely. Must we, for these markets, give up the system of causes already adopted? No, because it is possible to add to these causes one or more accessory prepositions which, playing the rôle of friction in mechanics, will make it possible to explain the newly observed phenomena. We say for this purpose that, "competition no longer has free play," that all the "goods are no longer identical," or that "the publicity given to sales is insufficient," and it is possible to deduce with the aid of the definitions and axioms already posited, the phenomena presented by the markets under consideration. The causes remain true.

Having thus demonstrated in all cases the law of supply and demand, we are going to generalize the results obtained by extending them to the theory of production. For the sake of greater simplicity, we shall put ourselves in a situation where only one

product is manufactured, thus necessitating the employment of only one class of capital and of a single class of labor under the direction of an entrepreneur. The equations which we set up can be generalized without difficulty for a case where the enterprise utilizes capital and labor of several varieties. We shall posit the following three axioms:

The quantity of labor offered to a given enterprise is an increasing function of the wages offered by the entrepreneur. Designating by  $L$  this quantity of labor and by  $w$  the wages, we may write

$$L = \psi(w) \qquad \psi(w) > 0$$

The quantity of capital offered a given enterprise is an increasing function of the interest which is paid. If  $c$  is the capital offered and  $i$  the interest, we have

$$c = \pi(i) \qquad \pi(i) > 0$$

Finally, the entrepreneur seeks to realize the maximum profit in all cases.

We shall now be able to set up equations of equilibrium of production by stating that the quantities of labor, of capital, and of product, offered during a unit of time, are equal to the quantities demanded, the profit of the entrepreneur being maximum.

By virtue of the preceding axioms we may write

$$(1) \qquad L = \psi(w)$$

$$(2) \qquad c = \pi(i)$$

Let us call  $l$  the quantity of labor necessary for the manufacture of a unit of manufactured goods (con-

stant coefficient depending at each instant upon the state of technique) and 0 the quantity of goods manufactured and therefore offered for sale. We may write the equation

$$(3) \quad l0 = L'$$

designating by  $L'$  the quantity of labor required by the entrepreneur.

$c$  representing similarly the quantity of capital necessary for the manufacture of a unit of manufactured goods, and  $C'$  the quantity of capital required by the entrepreneur, we may again write

$$(4) \quad c0 = C'.$$

This posited, let us assume that there is equilibrium in the market of capital and labor, supply being equal to demand. This gives us the two equations

$$(5) \quad L = L'$$

$$(6) \quad C = C'$$

We saw, furthermore, in a preceding paragraph that the demand for a product is a function of its price

$$(7) \quad D = \phi(p)$$

and we know that  $\phi'(p) < 0$  [ $\phi'(p)$  being the derivative of  $p$  which we have seen to be negative]. The equation

$$(8) \quad 0 = D$$

thus expresses the fact that the supply and demand of products are equal.



Finally the profit  $k$  realized by the entrepreneur is evidently the difference between the total income from the sale,  $0p$ , of the goods manufactured within the given unit of time and their net cost in capital and labor which is  $L'w + C'i$ .

$$(9) \quad k = 0p - L'w - C'i$$

Let us now state that this profit is maximum, or more exactly, the greatest possible. This gives us

$$dk = 0$$

that is

$$(10) \quad 0dp + pd0 - L'dw - wdl' - Cdi - idc' = 0$$

We thus obtain a system of 10 equations with 10 unknowns,  $L, w, C, i, L', C', 0, D, p, k$ . The system of the solutions of these 10 equations characterizes the state of equilibrium of production. The fact that this system is unique shows us that there exists a necessary relation between the different factors of production, salary, labor, and interest on capital, for examples, a connection which cannot be modified without the equilibrium being broken.

To utilize these equations, we are going to differentiate the first 8 and compare the differential equations thus obtained with the 10th. We obtain a system of 9 differential equations with 9 unknowns.

In order for them to be compatible the determinant of the unknowns must be zero. This determinant of

the 9th order develops itself easily enough and furnishes us equation 9 of the system below

- (1)  $L = \psi w$
- (2)  $c = \pi(i)$
- (3)  $lw = L'$
- (4)  $c0 = C'$
- (5)  $L = L'$
- (6)  $C = C'$
- (7)  $D = \phi(p)$
- (8)  $0 = D$
- (9)  $\psi'(w)\pi'(i)\{0 + \phi'(p)[p - lw - ci]$   
 $\quad - \phi'(p)[lL'\pi'(i) + cC'\psi'(w)] = 0$
- (10)  $K = 0p - L'w - C'i$

A knowledge of this system permits us to state precisely in advance, and to measure the effect of, any one of the functions which enter into it. A study of trade union rules thus permits us to determine the function  $L = \psi(w)$  and a study of the way capital is employed permits us to determine  $C = \pi(i)$ .

In a state enterprise in particular, the quantity of capital put at the disposal of the enterprise is independent of the rate of interest by which in private business remuneration of the capital employed is determined. Analytically, this condition is written

$$\pi'(i) = 0$$

and the equation (9) of the maximum profit becomes

$$(11) \quad \phi'(p)cC'\psi'(w) = 0$$

In the domain of state manufactures, the first member of the preceding equation is essentially

negative,  $\phi'(p)$  being negative,  $C'$  and  $\psi'(w)$  being positive for the usual values of prices and wages. The equation of the maximum profit cannot, therefore, be satisfied. In other words, an enterprise which does not draw its capital from a market governed by the law of supply and demand is unable to give its exploiters the maximum profit.

We thus rediscover by the deductive route a law that observation seems to confirm.

In order for equation (11) to be satisfied, it is necessary to have either  $\phi'(p) = 0$  or  $\psi'(w) = 0$ . In the first case, the demand for the products would have to be independent of the price, a condition realized only in a society whose products are distributed by means of authority. In the second, the supply of labor would have to be independent of the wages paid. This can only happen in the employment of military labor or, in general, by forced labor imposed on private individuals. We have thus shown the impossibility of going half-way along the path of communism which impossibility the perfect symmetry of equation (9) in  $L$  and  $C$  would have permitted us to foresee.

This example shows us the interest that a knowledge of a system of equations which represents the phenomenon of production can have.

But there is more. We are convinced that a careful study of statistics would make it possible, in many cases, to determine within the range of market equilibrium, the relative value of the functions

entering into the preceding system as well as those of its derivatives. We should then be able to foresee the effect of any measure determining *a priori* any one of the factors of production, the effect, for example, of a rise in wages resulting from a strike or of a regulation of prices determined either by foreign competition or by customs measures. It is unnecessary to emphasize the interest which such prognostications would have.

We believe that we have demonstrated in the preceding discussion the possibility of a mathematical political economy. In all the cases studied, we have found as many equations as unknowns, and in every case the theorems derived from our initial propositions (the definition of the magnitude of want, and the axiom of maximum utility) have expressed laws which observation of facts has confirmed. In the present state of our deductions, we may, therefore, affirm concerning our initial propositions, not that they are the best—only a further development of the theory will make it possible to judge of that—but that they are provisionally true.

We have proved also, and that was our purpose, that political economy is a science just as truly as is geometry or physics.

The importance of a mathematical theory thus constructed has often been questioned. We say that it is as great as that of any of the scientific theories previously studied. Mathematical political economy is, first of all, a logical and rigorous explanation of

phenomena known by experience. For this reason, it answers a need of our minds just as much as does the atomic theory. Furthermore, it makes possible the discovery of economic laws empirically observable, which, without it, would never have been isolated from the immense complexity of things. It is easier to verify the existence of a law surmised to be true, than to discover it when its existence is not suspected. Thus mathematical theory has permitted us to establish the form of the curve of supply as function of price, a form which observation of the facts confirms. On the other hand, when attention is not directed to this point, one is tempted to affirm that supply increases when the price increases, which is not always so.

Finally, it is often said that mathematical political economy is a science devoid of practical interest since it does not determine the functions whose existence it affirms. This objection does not seem to us to be justified for two reasons.

To show the existence of functions binding together certain magnitudes is, of itself, a result of great import. Physics derives results of considerable importance from affirming the existence of an equation characteristic of gases, even though unknown. But there is more to be said. Nothing proves to us that the function of  $f$ ,  $F$  and  $\psi$ , of which we have spoken in this chapter, cannot under given conditions be empirically determined. For this purpose we have at our disposal an infinite source of informa-

tion, namely, statistics and the figures of the different markets. Their study, systematically conducted with this end in view, would probably give exact indications about the form of these functions. We must remark in this connection that statistics are devoid of interest if the results to which they lead are not tied together by a theory of the kind we have sketched.

Finally, mathematical political economy makes known to us a series of propositions from which we know we can deduce all the economic laws known. It is, therefore, wise to affirm, in order to direct our actions, that there is a nature of things and that the propositions enunciated express its essence. It would be very surprising if such created causes, "true" in the sense which we have already stated, did not put into our hands a power comparable to that given to us by modern physics.

The affirmation that economic laws exist does not necessarily mean that we are their slaves. Weight exists and yet aeroplanes manœuvre in the air. We shall be able to derive from our knowledge of the laws of political economy a whole art, politics properly so-called, which will permit us to attain such ends as we may set up for ourselves. The art of politics will be to mathematical political economy what aeronautics is to physics.

For reasons already explained, empirical laws do not permit us to foretell the future, but only to foretell a set of conditions which will immediately follow



another fully determined set of conditions. We shall be able, therefore, by setting up certain conditions to bring about those results which we wish to bring about. It is only by knowing this law of succession that we are able to control events. For want of knowledge of this law and by legislating in a haphazard manner or according to emotion, results are produced radically unlike those sought. Hence the practical utility of Euclidean economic theories, *i. e.*, those which rediscover the laws of the world of experience.

#### B.—NON-EUCLIDEAN THEORIES OF POLITICAL ECONOMY

These are rational theories constructed from a group of premises of which one, at least, is in contradiction with one of the premises of a Euclidean theory. The theorems which are derived from it evidently do not coincide with the empirical laws which constitute the empirical reality of the moment.

The non-Euclidean theories of political economy are numerous. We shall take only one as an example, socialism under its most characteristic form, *i. e.*, that given to it by Karl Marx.

Instead of deriving economic laws from observation of the world of experience, he affirms *a priori* that their operation should have as its effect the equalization of the lot of all men. For this purpose, social organization should be such as to cause the disappearance of the inequality resulting from the difference in the education and fortunes of individuals



under the régime of profits and inheritance of property and from the unequal success of enterprises in an environment governed by free competition. Now collectivism, the inverse of communism, not assuring by means of authority the pure and simple distribution of all products necessary to life, must have recourse to a theory of value which will assure their distribution. As it refuses to recognize the existence of competition, an immediate consequence of the axiom of seeking maximum utility, it posits the bases of its theory *a priori*. It enunciates the axiom that the "value of a product is measured by the amount of social labor necessary for its production, *i. e.*, necessary in order for it to be produced by a workman of average ability, he using the best processes known at the time in question."<sup>1</sup>

From this axiom is derived a system of laws regulating the production and distribution of products, which laws are the counterpart of the empirical rules of Euclidean political economy. It is affirmed that, when the régime set forth has been realized, those laws will govern the succession of observable events.

Socialism thus conceived is a perfectly logical theory, thoroughly rigorous, but a theory which is not true in the sense which we have already made precise.

Let us see if it can become so some day. For this it would be necessary that the social régime under which we live be entirely overturned and that pro-

<sup>1</sup> C. Colson—Course in Political Economy.

duction and distribution be organized as the collectivists demand. Socialism, a logical theory, would then rediscover and explain the laws of the world of experience. It would become a Euclidean theory.

It is often said that socialism, or excessive intervention by the state, violates economic laws. That is an inaccurate statement. The advent of régimes different from ours would cause certain laws now governing phenomena to cease to operate, the conditions which they foretell no longer coming to pass. Competition being done away with, the law of supply and demand would no longer govern the succession of events but it would remain none the less true.

It is said also that socialism would, in conformity with known economic laws, diminish the yield of production. That is a result which is extremely probable. But barbarian invasions, wars and all social upheavals have also retarded what we have thought was the progress of humanity. They none the less came to pass because they were a normal drawing to a head of all the forces which make up life. Our ideas and our theories may be factors in the evolution of our universe. They do not determine it. It is the immense synthesis of all that is and all that has been that directs its march. Its end we do not know. Science confines itself to observing the reality of the moment and to deriving laws therefrom. It is in order to explain those laws that it constructs theories which are Euclidean today and may be no longer so tomorrow.

## CHAPTER XVI

### THE VALUE OF THE SOCIAL SCIENCES

Any study of the value of the social sciences must evidently consist of two parts, a study of the empirical laws which they enunciate and a study of the rational theories which make of those laws logical necessities.

All that has been said about the physical sciences is true here. The empirical laws of ethics or of political economy are facts and all that we create in these facts is the language which enunciates them. But we have shown that, by artificially isolating them in the aggregate of our sensations, we determined them to a great extent, and above all that here, as in mechanics, the choice of the unit of value itself determines both the form and the meaning of our empirical laws. We have shown that brute sensations do not necessarily obey the Law of Causality, but that the conventions of our methods of measurement impose that law upon the world.

Our empirical laws, nevertheless, are not arbitrary. They are, first of all, rules of action, and could be called, like those of Ahmes' geometry, "rules for acquiring wine in exchange for flour," or "clothing in exchange for pieces of gold."

The law of supply and demand or those which define our duties have, in fact, been ruling human life

for centuries. They are indeed a reality. All these laws, in the social as well as in the physical world, have a common character. They express a relation of succession, never a relation of causation. New laws which observation makes known to us seem incomprehensible so long as they have not been made to tie back to some existing theory and this cannot surprise us since these theories have been created to the sole end of explaining them. In fact, the mechanism in accordance with which the succession of events which these laws enunciate is brought to pass, is never given but is always created by us. And this explains why new empirical laws are always a great source of wonder.

This laid down, we believe we have shown that ethical theories, like all scientific theories, are only rational systems, shaped in order to transform the relations of succession we observe into relations of causation which we desire.

The propositions which serve as premises for these theories are causes in the logical sense of the word but causes created by us to the sole end of rediscovering deductively the laws which tie together the phenomena of experience. In any case, and this even more obviously in ethics than in the physical sciences, these causes, whether they be God, pleasure, or some imperative, are not given to us. They are posited *a priori* and their justification can be found only in the coincidence of the theorems derived there-

from with the corresponding rules empirically observed.

Here indeed is something which takes away much of the lustre of ethical theories. They have claimed to represent the profound nature of things, the first cause or the ultimate end of our world, yet we reduce them to merely convenient explanations of a reality which we do not understand. To say that such an ethical theory is true is an assertion devoid of meaning. All that we can claim is that, at a given moment, a moral or economic theory is good when it fully explains all the moral or economic facts known at the time under consideration. It is bad in the opposite case just as Euclidean geometry, good for beings unaware of Einstein's theory, becomes insufficient as soon as one knows the generalized theory of relativity.

One thus understands why the principal objection to the use of the deductive method in political economy is not sound. It is often said that the man we considered, summed up in the law of maximum utility, is not a real man; that psychology reveals to us an infinite variety of motives directing our action and that no single formula will ever render it possible for us to foretell individual conduct. In a word, it is charged that Axiom II does not represent "human nature."

All this is perfectly true but it is of no scientific interest. We have in no wise claimed to study men but only the appearances presented by an aggregate

of a great number of individuals. In order to explain them, we have created an "economic entity" (quite comparable to the molecules of the kinetic theory) whose characteristics are all contained in the definition of the magnitude of need, Axiom I and Axiom II. This sum-total of the created causes has interpreted the totality of phenomena of experience which the economic world presents. As these causes are not in contradiction with any known law or any existing theory, they are true in the usual sense of the word.

This point of view proves that there are social sciences existing by the same right as do the physical sciences, and that, like the latter, they are only a "creation of causes." They entail certain consequences which we are going to set forth.

The first is relative to the character of the theorems which the reasoning machine can derive from the causes thus posited. They constituting what is right must always be regarded as provisional. They are true only when observation has rediscovered in the world of experience laws which express them. Until then, they are only logical necessities, and become that again, ceasing to be true, when the laws which they rediscover no longer govern the succession of phenomena at the moment under consideration.

Thus is justified our assertion that social theories derive all their uncertainty from their indentity with geometry. Like all sciences, the social sciences can only advance by keeping in contact with reality, and



the bridge which assures this contact is the system of initial propositions, axioms and definitions. In order to make the theorems of our ethical or economic geometries coincide with the empirical laws observed, we have only one means at our disposal, *viz.*, to modify our initial propositions. That is what gives the art of definition all its value. That art is in a way the only physical part of scientific theories.

We can now understand the futility of efforts to justify the existing social state by rational theories. Moral or economic theories no more determine the form of our society than the kinetic theory determines the properties of gases. The social state exists, brought about by the total life in our universe, and our theories, Euclidean today and perhaps no longer so tomorrow, were created only *a posteriori* to rediscover its laws.

Thus is it understood that the political opinions of each individual are determined, with few exceptions, by the social class to which he belongs. In fact, each of us is led, according to temperament, fortune, birth, and material conditions of life, to regard as paramount a certain number of practical social laws, which, according to the person in question, should rule the world. The political opinions which one makes one's own are never more than a system of non-Euclidean causes designed to rediscover and to justify the rules which one would like to see adopted.

Here again we do not believe that our conception involves scepticism. With a knowledge of those



causes, which we have created and which permits us to foresee the succession of phenomena, we can act as if there were a nature of things and as if its true essence were known to us. The nature of things we "adopted," was not chosen at random, but for the precise purpose of facilitating our action. Furthermore, no more than can the chemist in his laboratory, can one who lives in the world escape the need for beliefs, which is what makes us assert the absolute truth of our theories. The coincidence between the results we rationally deduce and the laws we empirically observe is too perfect and too deep-seated to leave any room for doubt. Only the philosopher, in the moment of respite which he allows himself, can contemplate their true nature.

Our conception of social theories, though it makes no change in their practical value, is by contrast rich in consequences as to their study and exposition.

The fundamental part of all social sciences will be the search for empirical laws. The materials at our disposal to facilitate this quest are history, statistics, and average prices of all sorts. Their study, systematically conducted, alone can lead to the discovery of new laws or to the verification of rational laws supposed to be true.

To construct such theories, it is necessary to enunciate with the greatest clearness the axioms and definitions which are to serve as their bases. The rules of Pasch have to be rigorously observed. The next step is to derive from such initial proposi-

tions, by the deductive route, all the conclusions which they imply. Every time we thus arrive at a new theorem, we have to seek in nature for some empirical law to verify it.

On these conditions, and on these conditions only, can there be carried on the development of a theory which will be the truer the greater the number of its consequences which are capable of verification. Thus can we understand the progress of scientific theories which become instruments more and more efficacious, more and more certain.

We have followed that procedure in constructing the theory of political economy which we have outlined. We believe it could be followed also in ethics and that it would give interesting results even in aesthetics.

Systematically expounded under this form, many theories now admitted would turn out to be insufficient as an explanation of the facts, but several would prove to be good and among these one could make a choice. The best would obviously be that one which would utilize the least number of axioms and would best connect up with neighboring theories just as physics ties into mechanics.

We are persuaded that systematically adopting these points of view would lead to immense progress in all of the social sciences, whose existence and nature they make clear, and that such progress would put into our hands a power comparable to that given to us by modern physics.

## CHAPTER XVII

### CONCLUSIONS

We have studied, in the preceding pages, most of the human sciences. We have found that they are always, in the moral as well as in the physical world, a purely logical edifice which permits us, starting from axioms and definitions laid down *a priori*, to rediscover by the deductive process the aggregate of the empirical laws known at the moment under consideration. Thus in all cases, the building up of a science is truly a "creation of causes."

We are now in a position to understand what we mean when we assert that our sciences are true. The truth of the causes we have created is not a question. We have no means of ascertaining this one way or the other. We are, on the other hand, sure of the coincidence of the consequences they involve with the laws of the world of experience. That is the sole criterion of truth which we have.

The question as to whether the external world exists outside of ourselves, therefore, no longer comes up and no longer has any meaning, any more than has the question as to whether our logic is that of things. The "things" which are the subject-matter of science have been created by us, chosen so that by our logic we can derive from them laws which tie to-

gether the phenomena of experience. These laws constitute reality, all of it.

They are empirically known to us and yet we have seen that they were not imposed upon us. Our choice of the units in which they are expressed determines them to a large extent and impresses upon them this common character, namely, conformity with the Law of Causality.

Finally, we have seen that reality itself is not immutable. Improvement in our methods of observation and the evolution of life make it clear that laws, which now seem to us to regulate the succession of appearances, will not do so tomorrow. Theories which explain them will remain logical, but will cease to be Euclidean, and the causes upon which they were founded will no longer be true.

Thus life pursues its way. And the savant has constantly to make the infinite variety of things " thinkable " by creating causes.











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